Applied Mathematics 30

# Information Bulletin

## 2002–2003 School Year

## Diploma Examinations Program

Alberta LEARNING

103-05437/

This document was written primarily for:

| | |
|---|---|
| Students | ✓ |
| Teachers | ✓ Applied Mathematics 30 Teachers |
| Administrators | ✓ |
| Parents | |
| General Audience | |
| Others | |

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Please note that if you are attempting to access one of the direct web site links referred to in this document, and the link is not accessible, diploma examination-related materials can be found through the Alberta Learning web site address at www.learning.gov.ab.ca.

At the home page, click on the link *Kindergarten to Grade 12* then, click on the link *Provincial Testing*. Next, click on either *Diploma Examinations* or one of the specific links listed under the *Diploma Examinations headings*.

## *Bulletin Information*

The purpose of this bulletin is to provide students and teachers of Applied Mathematics 30 with information about the diploma examinations scheduled in the 2002–03 school year.

This bulletin includes descriptions of the Applied Mathematics 30 diploma examinations that will be administered in January, June, and August of 2003, descriptions of the *acceptable standard* and the *standard of excellence*, and subject-specific information. The mark awarded to a student on the Applied Mathematics 30 diploma examinations in 2003 will account for 50% of the student's final blended mark, and the school-awarded mark will account for the remaining 50%.

Teachers are encouraged to share the contents of this bulletin with students.

A Mathematics 33 diploma examination will continue to be available in all regular writing administrations until August 2003. These examinations will be secured (i.e., not released to educators, students, or the general public after the administration). The Mathematics 33 diploma examinations written in 2003 are intended for repeating students, adult students, distance learning students, and students with special circumstances.

For further information regarding program implementation, refer to the Alberta Learning web site at http://www.learning.gov.ab.ca.

## *Objectives of the Course*

The Applied Mathematics 30 course emphasizes the application and relevance of mathematics in daily life. In applied mathematics, numerical and geometrical methods are used to solve problems. Algebraic constructs are taught as needed during classroom investigations or data analysis. Tools such as graphing calculators and spreadsheet applications are commonly used to solve problems in applied mathematics.

Students are expected to communicate solutions to problems clearly and effectively when solving both routine and non-routine problems. Technology is to be used for exploration, modelling, and problem solving. Students are also expected to apply mathematical concepts and procedures to real-life problems.

## *Standards*

**Curriculum Standards**

Provincial curriculum standards help to communicate how well students need to perform in order to be judged as having achieved the learnings specified for Applied Mathematics 30. These specific statements of standards are written primarily to apprise Applied Mathematics 30 teachers of the extent to which students must know the Applied Mathematics 30 content and must demonstrate the required skills to be able to pass the examination.

**Examples of Questions**

In this bulletin, there are examples of questions that students should be able to answer in order to demonstrate the *acceptable standard* and the *standard of excellence*. The examples provided are by no means exhaustive; they are intended to provide a profile of acceptable and excellent achievement. Some examples and solutions were developed and validated by classroom teachers of mathematics but have not been validated with students. Other examples were taken from previous diploma examinations. All examples model the types of questions and problems that students should be able to solve and the types of activities that students should be able to perform in order to meet the specific outcome to which the questions are linked.

## *Achievement Standards*

**Acceptable Standard**

Students who attain the *acceptable standard* but not the standard of excellence in Applied Mathematics 30 will receive a final course mark between and including 50% and 79%. Typically, these students have gained new skills and knowledge in mathematics and they can apply mathematical concepts and procedures to find a solution to routine problems. They have demonstrated mathematical skills and knowledge in the six topics of the Applied Mathematics 30 curriculum and exhibit an ability to apply a broad range of problem-solving skills to these content strands.

**Standard of Excellence**

Students who attain the *standard of excellence* will receive a final course mark of 80% or higher. Such students have demonstrated their ability and interest in mathematics and have confidence in their mathematical skills. These students can choose the most efficient method for solving problems. They can also find more than one solution and can solve non-routine problems.

At a provincial level, it is expected that at least 85% of students will achieve a final course mark of 50% or higher and that at least 15% of students will achieve a final course mark of 80% or higher.

## *Term Project*

Alberta Learning will produce two projects (large-scale mathematical problems) per year for Applied Mathematics 30. These projects are designed to be completed in three to five hours of student time. Use of these projects is optional. Teachers may choose to use the projects as part of their assessment. A sample solution and scoring rubric will be provided for each project. One of the written-response questions worth 10% on the diploma examination will be related to its corresponding project. The first project will be distributed to schools in mid-August 2002, and one of the written-response questions on the January 2003 diploma examination will be related to this project. The second project will be distributed to schools in mid-January 2003, and one written-response question on each of the June 2003 and August 2003 diploma examinations will be related to this project. Students who did not complete the project but who have completed the course will have the knowledge to answer the written-response question; however, students who have completed the project will have had experience with the related mathematical skills in another context.

## *Examination Specifications*

Each Applied Mathematics 30 diploma examination is designed to reflect the core content outlined in the *Applied Mathematics 30 Program of Studies*. The examination is limited to those expectations that can be measured by a paper-and-pencil test. Therefore, the percentage weightings shown below will not necessarily match the percentage of class time devoted to each unit. The examination is developed to be completed in 2.5 h; however, students may take an additional 0.5 h to complete the examination.

The content for the Applied Mathematics 30 Diploma Examinations in the 2002–03 school year is emphasized as follows.

| ***Machine-Scored Content*** | ***Percentage Emphasis*** |
|---|---|
| Matrices and Pathways | 17 |
| Finance and Spreadsheets | 17 |
| Statistics and Probabilities | 17 |
| Vectors | 18 |
| Cyclic, Recursive and Fractal Patterns | 18 |
| Design | 13 |
| ***Multiple-Choice and Numerical-Response Questions*** | 65 |
| ***Written-Response Questions*** | 35 |

Written-response questions assess the degree to which students can draw on their mathematical experiences to solve problems and explain mathematical concepts. Therefore, the written-response questions will not necessarily fall into a particular unit of study but may cover more than one unit or may require students to make the connections between mathematical concepts. Of the seven mathematical processes, the written-response questions will address communication, connections, reasoning, problem solving, and technology.

Procedural, conceptual, and problem-solving cognitive levels are addressed throughout the examination. The approximate emphasis of each cognitive level is given below.

| ***Multiple Choice, Numerical Response, and Written Response*** | ***Percentage Emphasis*** |
|---|---|
| Procedures | 35 |
| Concepts | 30 |
| Problem Solving | 35 |

Since the machine-scored and written-response questions are weighted components, a student's examination mark can be calculated using the following equation.

$$\left(\frac{x}{39}\times 65\right)+\left(\frac{y}{10}\times 20\right)+\left(\frac{z}{5}\times 15\right)=\text{exam mark}$$

$x$ = Student's total score on multiple-choice and numerical-response questions
$y$ = Student's total score on the two 10% written-response questions
$z$ = Student's score on the 15% written-response question.

## *Examination Design*

The design of the 2002–03 Applied Mathematics 30 diploma examinations is as follows:

| *Question Format* | *Number of Questions* | *Percentage Emphasis* |
|---|---|---|
| Multiple Choice | 33 | 55 |
| Numerical Response | 6 | 10 |
| Written Response | 3 | 35 |

The six numerical-response questions are interspersed throughout the multiple-choice questions, according to content topic.

The three written-response questions are arranged in the examination in the following manner:

- two questions, each worth 10% of the examination, are interspersed with the machine-scored questions, according to content topic
- one question, worth 15% of the examination, is the last question on the examination

Information required to answer multiple-choice and/or numerical-response questions is often located in a box preceding the question. The number of questions that require the use of the information given in the box will be clearly stated above the box; e.g., "*Use the following information to answer the next two questions.*"

**Multiple Choice and Numerical Response**

For multiple-choice questions, students are to choose the correct or best possible answer from the four alternatives.

For some numerical-response questions, students are to calculate a numerical answer and record their answer in a separate area of the answer sheet. When the answer to be recorded cannot be a decimal value, students are asked to determine a whole number value (e.g., the number of people is _____, the degree of this polynomial is _____.) If the answer can be a decimal value, then students are asked to record their answer correct to the nearest tenth or nearest hundredth, as specified in the question.

Other numerical-response questions require students to record their understanding of a conceptual idea. The following is an example of such a question.

*Use the following information to answer the next question.*

$$2\begin{bmatrix} 1 & 0.5 \\ 1.5 & 4 \end{bmatrix} = \begin{bmatrix} a & b \\ c & 8 \end{bmatrix}$$

**Numerical Response**

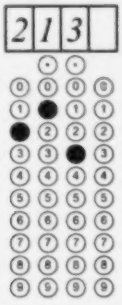
**6.** In the equation above, the value of

***a*** is _____ (Record in the **first** column)
***b*** is _____ (Record in the **second** column)
***c*** is _____ (Record in the **third** column)

(Record all **three digits** of your answer in the numerical-response section on the answer sheet.)

**Answer: 213**

**Record 213 on the answer sheet** →




**Written Response**

The written-response questions focus on students' understanding of the process of solving a problem and encourage students to take risks to arrive at a solution. Students will be rewarded for selecting a problem-solving strategy and for carrying through with the strategy to find a solution. The written-response questions focus on the students' use of mathematical processes. To achieve the *standard of excellence*, students must be able to select a strategy, carry it through, and solve the problem correctly. The written-response questions also focus on students' understanding of mathematical concepts and allow for flexibility in evaluating students' communication and problem-solving abilities in mathematics.

Written-response questions ask students to solve, explain, justify, or prove their solution.

In scoring the written-response questions, markers will evaluate how well students

- understand the problem or the mathematical concept
- correctly use the mathematics
- use problem-solving strategies and explain their answer and procedures
- communicate their solutions and mathematical ideas

One of the two 10% questions will be related to the term project provided to all schools by Alberta Learning. The 15% question will focus on the mathematical processes of reasoning and connections.

Above all, students should be encouraged to try to solve all problems. Even an attempt at a solution could be worth some marks. The three written-response questions are each scored with a five-mark scoring rubric. Students should note that markers will be instructed to consider appropriate use of units and appropriate rounding of answers in all solutions to written-response questions. A student will not be penalized more than once for errors of this type.

## *Assessment of the Mathematical Processes*

Open-ended questions provide a way in which to assess mathematics as a common human activity. Open-ended questions allow students to communicate a response by asking them to explain their reasoning, explain their solution, describe mathematical situations, write directions, create new problems, create new strategies, generalize a mathematical situation, and formulate hypotheses.

The written-response questions provide an opportunity to evaluate several mathematical processes as outlined in the *Program of Studies*: communication, connections, estimation and mental mathematics, problem solving, reasoning, technology, and visualization. One of the 10% written-response questions will focus on problem solving, and the other 10% question will focus on communication. The 15% written-response question will focus on connections and reasoning. Technology may be incorporated into all three written-response questions.

**Communication**

"Students need to communicate mathematical ideas clearly and effectively, orally and in writing." (*Mathematics Applied and Pure Programs,* page 7) It is expected that students, in answering the written-response questions, will provide clear, concise answers. Students should clearly communicate all pertinent steps used to arrive at a solution. Students should use correct mathematical notation and conventions. Graphs or diagrams may be presented in solutions.

In communicating an answer, students must be aware of the degree of accuracy required as well as the appropriate units involved.

**Problem Solving**

"Problem solving is the focus of mathematics at all grade levels. The development of each student's ability to solve problems is essential. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts." (*Mathematics Applied and Pure Programs,* page 9) Approximately 36% of the machine-scored questions as well as one of the written-response questions will be related to a problem-solving context. It is expected that students will be able to select an appropriate problem-solving strategy to find the solution to a given problem or to model real-world problem situations.

Students should not expect questions on a particular concept to be asked in the same manner every time. They must be able to adapt to changes in the format, which will help to improve their problem-solving skills.

**Connections**

Students need numerous and varied experiences to appreciate the usefulness of mathematics. They must explore connections between different areas of mathematics and between mathematics and other disciplines. The "connections process" also includes relating mathematics to their own daily experiences. "When mathematical ideas are connected to each other through concrete, pictorial, and symbolic representations, students begin to view mathematics as an integrated whole." (*Mathematics Applied and Pure Programs,* page 8)

The connections process is often linked with problem solving and reasoning, as it is an application of these other processes.

**Reasoning**

"Students need to develop confidence in their ability to reason and to justify their thinking inside and outside of mathematics. The power of reasoning helps students to make sense of mathematics, to make logical inferences from given information, and to convince others of a mathematical meaning.

"Inductive reasoning helps students explore and make conjectures from activities that allow generalizations from a pattern of observations."

"Deductive reasoning helps students test conjectures and build arguments that serve to validate logical thinking. Deductive reasoning also assembles a structured body of knowledge." (*Mathematics Applied and Pure Programs,* page 10)

Reasoning allows students to interpret mathematics and aids in the choice of appropriate problem-solving strategies. Students may be asked to demonstrate logical reasoning when judging the validity of arguments, testing conjectures, and constructing valid arguments.

**Technology**

"Electronic technologies—calculators and computers—are tools that furnish visual images of mathematical ideas, facilitate organization and analysis of data, and support investigation by students. They allow students to focus on decision making, reflection, reasoning, and problem solving. The computational capacity of technological tools extends the range of problems accessible to students as well as enabling them to execute routine procedures quickly and accurately, thus allowing more time for conceptualizing and modelling. Technology also blurs some of the separations among topics in algebra, geometry, and data analysis by allowing students to connect knowledge and ideas from one area of mathematics to another." *(Principles and Standards for School Mathematics, NCTM 2000,* pages 24 to 25*)*

## *General Scoring Guide*

**Scoring Guide for Written-Response Questions**

Student responses to written-response questions will be scored against specific question scoring rubrics based on the *General Scoring Guide*. Credit will be given to students who appropriately demonstrate unusual insight in addressing the question.

This scoring guide reflects a mark based on four dimensions:

- mathematical understanding
- application of processes
- clarity of communication
- use of technology

| GENERAL SCORING GUIDE | |
|---|---|
| **1 mark** | In the response, the student<br>• applies some relevant mathematical knowledge to explore the initial stages of the problem; however, the response reflects a misunderstanding of the problem<br>• uses a relevant strategy, mathematical process, or problem-solving technique to explore the initial stages of the problem<br>• communicates very little relevant information and lacks clarity<br>• uses technology inappropriately or the use of technology is not evident |
| **2 marks** | In the response, the student<br>• applies some relevant mathematical knowledge to find partial solutions to the problem; however, the response reflects a minimal understanding of the problem<br>• uses relevant strategies, mathematical processes, or problem-solving techniques to find a partial solution to the problem<br>• communicates strategies in a manner that lacks clarity or is incomplete<br>• uses technology where appropriate; however, errors are evident |
| **3 marks** | In the response, the student<br>• applies mathematical knowledge to find partial solutions to the problem and reflects a basic understanding of the problem<br>• uses appropriate strategies, mathematical processes, and problem-solving techniques to find partial solutions to the problem<br>• communicates strategies and solutions in an organized manner; however, errors, inconsistencies, and omissions affect clarity<br>• uses technology appropriately; however, there are inconsistencies in their application |
| **4 marks** | In the response, the student<br>• applies appropriate mathematical knowledge to find a complete solution to the problem and reflects a good understanding of the problem<br>• uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete solution to the problem; however, the solution contains an error that hinders understanding of the response<br>• communicates strategies and solutions in an organized manner; however, errors or omissions may affect clarity<br>• uses technology appropriately |
| **5 marks** | In the response, the student<br>• applies appropriate mathematical knowledge to find a complete and correct solution to the problem and reflects an excellent understanding of the problem<br>• uses appropriate strategies, mathematical processes, and problem-solving techniques to find a complete, correct solution; the solution may have a minor error but it does not hinder the understanding of the response<br>• communicates strategies and solutions in a clear, complete, and organized manner that reflects a thorough understanding of the problem<br>• uses technology effectively |

# *Scoring Written-Response Questions*

This section provides examples of selected written-response questions, sample student responses, and scoring rationales as they relate to the general scoring guide. These examples are intended to inform teachers and students of how the scoring guide is applied to specific questions and to encourage the use of the general scoring guide in class assignments.

## Sample 1

*Use the following information to answer the next question.*

A group of biologists studied the population of wolves in an area in northern Canada. The biologists found that the number of wolves was directly related to the number of caribou found in the study area. The wolf population for the period of the study is shown in the table below.

| Year | Wolf Population |
|---|---|
| Base year (0) | 400 |
| 1 | 548 |
| 2 | 800 |
| 3 | 1 168 |

**Written Response—10%**

2. **a.** State the exponential regression equation for this data in the form $y = ab^x$. Round the value of $a$ to the nearest whole number and the value of $b$ to the nearest hundredth.

$y = (392)(1.43)^x$

**b.** The biologists found that a herd of approximately 17 800 caribou was needed to sustain 650 wolves. If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how many caribou would be required to sustain the wolf population in year 4?

$y = (392)(1.43)^4$

$y = 1639$

*Use the following additional information to answer the next part of the question.*

The area the biologists studied was approximately 14 500 km$^2$, and they found the maximum population density of wolves to be 18 wolves/100 km$^2$.

**c.** • What is the maximum number of wolves that this area can sustain?

14500 km$^2$
× 18

805 wolves/100km$^2$

If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how much time will have elapsed when this maximum number of wolves is reached?

2 years

### Score – 2

### Rationale

In the response, the student applies some relevant mathematical knowledge to find partial solutions; however, the response reflects a minimal understanding of the problem. The student's answers are communicated in a manner that lacks clarity and are supported with minimal details. The student has provided some correct answers but fails to clearly connect the answers to the context of the problem.

## Sample 2

*Use the following information to answer the next question.*

A group of biologists studied the population of wolves in an area in northern Canada. The biologists found that the number of wolves was directly related to the number of caribou found in the study area. The wolf population for the period of the study is shown in the table below.

| Year | Wolf Population |
|---|---|
| Base year (0) | 400 |
| 1 | 548 |
| 2 | 800 |
| 3 | 1 168 |

**Written Response—10%**

2. a. State the exponential regression equation for this data in the form $y = ab^x$. Round the value of $a$ to the nearest whole number and the value of $b$ to the nearest hundredth.

$y = 392 \times 1.43^x$

b. The biologists found that a herd of approximately 17 800 caribou was needed to sustain 650 wolves. If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how many caribou would be required to sustain the wolf population in year 4?

$y = 467796 \times 100^x$ equation for relation of caribou and wolves

$y = 467796 \times 100^4$

$= 4678$ caribou

*Use the following additional information to answer the next part of the question.*

The area the biologists studied was approximately 14 500 km$^2$, and they found the maximum population density of wolves to be 18 wolves/100 km$^2$.

c. What is the maximum number of wolves that this area can sustain?

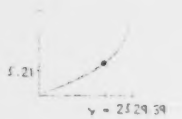
$$\frac{18w}{100\ km^2} = \frac{x}{14500\ km^2}$$

$x = 18w \times 14\,500\ km^2$

$= 2610$ wolves

If the wolf population continued increas ing at the same rate as it did in years 0 to 3 of the study, how much time will have elapsed when this maximum number of wolves is reached?

$x = 1392 \times 1.43^x$




In about 5.25 years the population will have reached its max

## Score – 3

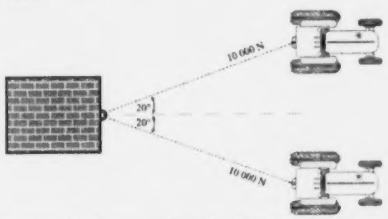
## Rationale

By calculating the exponential regression equation, this student demonstrates an appropriate use of technology, but there are inconsistencies in its application. The student failed to correctly apply the exponential regression equation to find the number of wolves (and therefore the number of caribou) in year 4. Since the student was unable to use the regression equation appropriately, it is unclear whether the student had a good understanding of the problem. The remainder of the work shows a basic understanding.

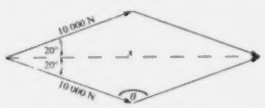
## Sample 3

*Use the following information to answer the next question.*

Two tractors pull on a large block with a force of 10 000 N each. Each of the tractors pulls at an angle 20° from the direction that the block is to move in, as shown below.




The force exerted by the tractors can be modelled by the vector diagram shown below.




**Written Response—10%**

1. a. Determine the measure of angle $\theta$ in the vector diagram.

40 + 40 + 80 180 - 80 = 100

$\frac{100}{2} = 50$

θ = 50°

b. Calculate the magnitude, $x$, of the resultant force that the tractors exert on the block.

$$\frac{a}{\sin A} = \frac{b}{\sin b}$$

$$\frac{x}{\sin 50°} = \frac{10000}{\sin 20}$$

$$x = 22\,398 \text{ N}$$

c. If a force of 18 000 N is required to move the block, will the tractors be able to do so? Explain and justify your answer mathematically.

Yes because the resultant force of the tractors is 22 398 N which is well over 18 000 N

## Score – 3

## Rationale

The student's response reflects a basic understanding of the problem rather than a good understanding as a result of one misconception. The student has indicated, through the work shown in part a, a belief that the sum of the interior angles in a parallelogram is 180° instead of 360°. Since this error is a conceptual misunderstanding rather than a calculation error, the student earned 3 marks out of a possible 5.

## Sample 4

*Use the following information to answer the next question.*

A group of biologists studied the population of wolves in an area in northern Canada. The biologists found that the number of wolves was directly related to the number of caribou found in the study area. The wolf population for the period of the study is shown in the table below.

| Year | Wolf Population |
|---|---|
| Base year (0) | 400 |
| 1 | 548 |
| 2 | 800 |
| 3 | 1 168 |

**Written Response—10%**

2. a. State the exponential regression equation for this data in the form $y = ab^x$. Round the value of $a$ to the nearest whole number and the value of $b$ to the nearest hundredth.

$y = a \times b \wedge x$

$y = 392 \times 1.43 \wedge x$

b. The biologists found that a herd of approximately 17 800 caribou was needed to sustain 650 wolves. If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how many caribou would be required to sustain the wolf population in year 4?

In the 4th year the wolf population will be 1651, which is 483 more wolves than the previous year. The herd would nearly have to be doubled. Maybe 35000 caribou would be needed to sustain the wolf population.

*Use the following additional information to answer the next part of the question.*

The area the biologists studied was approximately 14 500 km$^2$, and they found the maximum population density of wolves to be 18 wolves/100 km$^2$.

c. •What is the maximum number of wolves that this area can sustain?

$\frac{14500\ km^2}{100\ km^2} = 145$

$145 \times 18$ wolves $= 2610$

The area can sustain 2610 wolves

If the wolf population continued increas ing at the same rate as it did in years 0 to 3 of the study, how much time will have elapsed when this maximum number of wolves is reached?

2610

The maximum number of wolves will be reached in exactly 5.274 years

$x = 5.274 \quad y = 2610$

## Score – 4

## Rationale

In the response, the student shows a good understanding of the problem, and uses appropriate strategies, mathematical process, and problem-solving techniques to find a complete solution. However, the student makes an error in part b by poorly estimating the required number of caribou. Because the student supplies minimal justification for the estimate, this error hinders understanding of the response.

## Sample 5

*Use the following information to answer the next question.*

A group of biologists studied the population of wolves in an area in northern Canada. The biologists found that the number of wolves was directly related to the number of caribou found in the study area. The wolf population for the period of the study is shown in the table below.

| Year | Wolf Population |
|---|---|
| Base year (0) | 400 |
| 1 | 548 |
| 2 | 800 |
| 3 | 1 168 |

**Written Response—10%**

2. **a.** State the exponential regression equation for this data in the form $y = ab^x$. Round the value of $a$ to the nearest whole number and the value of $b$ to the nearest hundredth.

| L1 | L2 |
|---|---|
| 0 | 400 |
| 1 | 548 |
| 2 | 800 |
| 3 | 1168 |

$y = 392 \times 1.43^x$

**b.** The biologists found that a herd of approximately 17 800 caribou was needed to sustain 650 wolves. If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how many caribou would be required to sustain the wolf population in year 4?

$$\frac{17800}{x} = \frac{650}{1639}$$

$$650x = 17800 \times 1639$$

$$\frac{650x}{650} = \frac{29174200}{650}$$

$$x = 44883.38$$

In year 4, to sustain the wolf population, 44883 caribou would be needed

*Use the following additional information to answer the next part of the question.*

The area the biologists studied was approximately 14 500 km², and they found the maximum population density of wolves to be 18 wolves/100 km².

**c.** •What is the maximum number of wolves that this area can sustain?

$14500 \div 100 = 145$

$145 \times 18 = 1610$

The maximum number of wolves that this area can sustain is 2610

If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how much time will have elapsed when this maximum number of wolves is reached?

(Used trace feature on calc to figure this out)

5.3 years will have elapsed when the maximum number of wolves is reached

## Score – 4

## Rationale

The student applies appropriate mathematical knowledge to find a complete solution to the problem. This student has communicated strategies and solutions in an organized manner; however, the student made a calculation error in part b. This error was carried through to part c but appropriately used to justify the response.

# Applied Mathematics 30

# Curriculum Standards and Example Questions

## 2002–2003

The Applied Mathematics 30 Curriculum Standards document is intended to assist teachers in implementing Applied Mathematics 30.

Some of the examples shown in this document were chosen to illustrate the intent of the particular outcomes; however, they will not necessarily be assessed on a diploma examination in the manner shown. These examples were developed and validated by classroom teachers of mathematics but have not been validated with students. Other examples are taken from previous Applied Mathematics 30 diploma examinations.

To meet the outcomes of Applied Mathematics 30, students will need access to an approved graphing calculator and a computer with a spreadsheet program. In most classrooms, students will use a graphing calculator on a daily basis. Refer to the Alberta Learning web site www.learning.gov.ab.ca and click *Provincial Testing* or page 128 of this document for a list of approved graphing calculators.

If you have comments or questions regarding this document, please forward them by e-mail to Deanna Shostak (Deanna.Shostak@gov.ab.ca) or Corlene Balding (Corlene.Balding@gov.ab.ca) phone (780) 427-0010 or fax (780) 422-4454

The Learner Assessment Branch would like to recognize and thank the many teachers throughout the province who helped prepare this document. We would also like to thank the Curriculum Branch and the Learning Technologies Branch for their input and assistance in reviewing these standards.

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Applied Mathematics 30 Formula Sheet 123

Applied Mathematics *z*-sore Table 124

## Matrices and Pathways

### *General Outcomes*

Describe and apply operations on matrices to solve problems, using technology as required.

Solve problems based on the counting of sets, using techniques such as the fundamental counting principle, permutations and combinations.

**General Notes:**

- A node is an intersection of two or more pathways on a grid.
- The intent of the matrices section is that students will recognize situations in which matrices may be applied. This section should not be an in-depth study of matrix operations. However, students will need to understand the basic operations and procedures of addition, subtraction, and multiplication of matrices in order to solve problems in a given context.
- Inverse matrices, determinants, and matrix solutions to linear systems are all beyond the scope of Applied Mathematics 30.
- Paper and pencil calculations involving operations on matrices should be limited to those that can be expressed as a matrix no larger than 3 by 3. Technology should be used for matrices of a larger size.

### *Specific Outcomes*

**Specific Outcome 1.1**

Solve pathway problems, interpreting and applying any constraints. [PS, R]

**1.1 Notes:**

- A formal study of permutations and combinations is beyond the scope of Applied Mathematics 30.
- The purpose of this outcome is to have students solve pathway problems by recognition and application of a pattern rather than by inspection alone.
- Tree diagrams and other organizational tools (e.g., Pascal's triangle) should be used to present solutions.

*Refer to examples 10, 11, and 12.*

**Specific Outcome 1.2**

Use the fundamental counting principle to determine the number of different ways to perform multistep operations. [PS, R]

**1.2 Note:**

- Questions should be kept at a level where students who achieve the *acceptable standard* can be successful.

*Refer to example 1.*

**Specific Outcome 1.3**

Perform, using technology only for larger matrices, the matrix operations of addition, subtraction, matrix multiplication and multiplication by a scalar [C, E, R, T, V].

**1.3 Note:**

- Paper and pencil calculations involving operations on matrices should be limited to those that can be expressed as a matrix no larger than 3 by 3.

*Refer to examples 2, 5, 6, 9, 13, and 14.*

**Specific Outcome 1.4**

Model and solve consumer and network problems, performing matrix operations and using algebraic solution strategies as needed. [CN, PS, T, V]

**1.4 Note:**

- Problems involving operations on matrices should be limited to those that can be expressed as a matrix no larger than 4 by 4.

*Refer to examples 3, 4, 7, 8, and 13.*

| *Acceptable Standard* | *Standard of Excellence* |
|---|---|
| The student can | The student can also |
| • solve simple pathway problems by inspection or recognition of a pattern | • recognize and apply a pattern as a solution to a pathway problem |
| • use the Fundamental Counting Principle to determine the number of different ways to perform multistep operations | • use the fundamental counting principle to determine the number of different ways to perform a series of multistep operations |
| • build a matrix from a given table | • build a matrix from a given context and solve problems associated with it |
| • recognize and produce a row matrix and column matrix | |
| • describe the dimensions of a matrix | |
| • identify conditions required to perform operations on matrices | |
| • identify the required matrix operation for a given context | |
| • perform operations of addition, subtraction, matrix multiplication, and scalar multiplication on matrices | |
| • identify components of a matrix or a matrix operation as they relate to a specific problem | |
| • make modifications to one or two components of a matrix in response to new scenarios | • make modifications to a matrix in response to new scenarios |
| • model a simple problem with a matrix | • model a complex problem with a matrix |
| • solve problems in which the matrix is given | |
| • participate in and contribute toward the problem-solving process for problems that require the analysis of matrices and pathways studied in Applied Mathematics 30 | • complete the solution to problems that require the analysis of matrices and pathways studied in Applied Mathematics 30 |

## *Examples*

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

*Use the following information to answer the next question.*

The Hawaiian alphabet has only twelve letters:

| | |
|---|---|
| Vowels | *a, e, i, o, u* |
| Consonants | *h, k, l, m, n, p, w* |

**1.** The number of 3-letter "words" that can be made using the Hawaiian alphabet if every "word" must have the pattern of consonant-vowel-consonant, and if letters can be repeated, is

**A.** 19

**B.** 175

**C.** 245

**D.** 1 728

**Solution:**

There are 7 choices for the first letter, 5 choices for the second letter and 7 choices for the last letter. Using the fundamental counting principle, there are 245 different 3-letter words that can be made.

**2.** On which of the following matrices can addition be performed?

**A.** $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} g \\ h \end{bmatrix}$

**B.** $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix}$

**C.** $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

**D.** $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$ and $\begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix}$

**Solution:**

In order to add two matrices, they must contain the same number of rows and columns. Therefore, the correct response is D.

*Use the following information to answer the next question.*

The table below shows the number of vehicles parked in a downtown parking lot over a three-day period.

| | Cars | Buses | Bicycles |
|---|---|---|---|
| Thursday | **72** | **6** | **7** |
| Friday | **81** | **2** | **2** |
| Saturday | **94** | **3** | **12** |

The charge for cars is \$6, the charge for buses is \$15, and there is no charge for bicycles. The total revenue for each of the three days can be determined from the product of the two matrices below.

$$\begin{bmatrix} 72 & 6 & 7 \\ 81 & 2 & 2 \\ 94 & 3 & 12 \end{bmatrix} \times \begin{bmatrix} 6 \\ 15 \\ 0 \end{bmatrix}$$

**3.** From this matrix operation, it can be determined that the revenue from

**A.** cars on Thursday was \$522

**B.** all vehicles on Saturday was \$609

**C.** all vehicles on Thursday was \$516

**D.** buses over the three days was \$516

**Solution:**

$$\begin{bmatrix} 72 & 6 & 7 \\ 81 & 2 & 2 \\ 94 & 3 & 12 \end{bmatrix} \times \begin{bmatrix} 6 \\ 15 \\ 0 \end{bmatrix} = \begin{bmatrix} 522 \\ 516 \\ 609 \end{bmatrix}$$

This matrix operation shows that the revenue from all vehicles on Thursday was \$522, on Friday was \$516, and on Saturday was \$609. Therefore, the correct response is B.

*Use the following information to answer the next two questions.*

Three major countries produce cars to be purchased in Canada: Canada, Japan, and Germany. A poll of car owners in Canada revealed that of people who presently own a Canadian-produced car, 51% would purchase another Canadian-produced car the next time they purchase a car. Of people who presently own a Japanese-produced car, 30% would purchase a Canadian-produced car the next time they purchase a car.

The following matrix shows detailed results of the poll.

| | CA | JA | GR | Other |
|---|---|---|---|---|
| CA | 0.51 | 0.32 | 0.12 | 0.05 |
| JA | 0.30 | 0.50 | 0.12 | 0.08 |
| GR | 0.35 | 0.15 | 0.40 | 0.10 |
| Other | 0.20 | 0.25 | 0.15 | 0.40 |

**4.** The entry in row 3 and column 2 indicates that

**A.** 15% of people who presently own a Japanese-produced car would purchase a German-produced car the next time they purchase a car

**B.** 12% of people who presently own a German-produced car would purchase a Japanese-produced car the next time they purchase a car

**C.** 15% of people who presently own a German-produced car would purchase a Japanese-produced car the next time they purchase a car

**D.** 12% of people who presently own a Japanese-produced car would purchase a German-produced car the next time they purchase a car

**Solution:**

The entry in row 3 and column 2 is 0.15. This represents the percentage of people who presently own a German-produced car, but would purchase a Japanese-produced car the next time they purchase a car.

*Use the following information to answer the next question.*

**A Matrix Equation**

$$\begin{bmatrix} -3 & 11 \\ 2 & 8 \end{bmatrix} + 3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 0 & 26 \\ 35 & 14 \end{bmatrix}$$

SE **Numerical Response**

**5.** In this equation, the value of $c$ is _____.

(Record your answer in the numerical-response section on the answer sheet.)

**Solution:**

$$\begin{aligned} 2 + 3c &= 35 \\ 3c &= 33 \\ c &= 11 \end{aligned}$$

SE **6.** Solve for $x$ in the following matrix.

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 6 \\ 1 & 2 & 8 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ x & 5 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 33 & 26 \\ 53 & 53 \\ 40 & 21 \end{bmatrix}$$

**Solution:**

**Method One** (Algebraic)

$$\begin{aligned} 2(2) + 3(x) + 5(4) &= 33 \\ 4 + 3x + 20 &= 33 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

**Method Two** (Inverse matrix)

$$\begin{bmatrix} 2 & 3 & 5 \\ 4 & 7 & 6 \\ 1 & 2 & 8 \end{bmatrix}^{-1} \times \begin{bmatrix} 33 & 26 \\ 53 & 53 \\ 40 & 21 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 5 \\ 4 & 1 \end{bmatrix}$$

Therefore, the value for $x$ is 3.

**Note:** Method Two is a recognizable method for solving this equation; however, questions that **require** inverse matrices are beyond the scope of Applied Mathematics 30.

7. The matrix below shows the time, in hours, that each of three toys—a car, a truck, and a motorcycle—spend at each of three factory assembly stations, A, B, and C. The "finishing time" for a given toy is the time required for it to pass through all three stations.

$$\begin{array}{r} \\ \text{Car} \\ \text{Truck} \\ \text{Motorcycle} \end{array} \begin{array}{c} \begin{array}{ccc} \text{A} & \text{B} & \text{C} \end{array} \\ \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

**a.** A toy car, truck, and motorcycle pass through station A. What is the total time that the three toys spend at station A?

**b.** What is the "finishing time" required for a truck?

**c.** Changes to the truck's design make it necessary to increase its finishing time by 20%. If the time spent on the truck at each station is increased by the same percentage, what is a matrix operation that will model this change?

**Solution:**

**a.** The total time spent at station A is the sum of the elements of the first column in the matrix

$$\begin{bmatrix} 0.7 \\ 0.6 \\ 0.4 \end{bmatrix}$$

$0.7 + 0.6 + 0.4 = 1.7$

The total time that the three toys spent at station A is 1.7 hours.

**b.** The finishing time for a truck is the sum of the elements of the second row of the matrix

$[0.6 \ \ 0.4 \ \ 0.5]$

$0.6 + 0.4 + 0.5 = 1.5$

A truck's finishing time is 1.5 hours.

**c.** **Method One**

Strip out the row that indicates the time the truck requires at each station and make it into a row matrix. This matrix can be multiplied by a scalar of 1.2.

$1.2[0.6 \ \ 0.4 \ \ 0.5] = [0.72 \ \ 0.48 \ \ 0.6]$

After the design changes, the truck will require 0.72 hours at station A, 0.48 hours at station B, and 0.6 hours at station C.

**Method Two**

Perform the following matrix multiplication.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.6 & 0.4 & 0.5 \\ 0.4 & 0.3 & 0.4 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.3 & 0.6 \\ 0.72 & 0.48 & 0.6 \\ 0.4 & 0.3 & 0.4 \end{bmatrix}$$

The time that the car and motorcycle spend at each station has not changed, but the time that the truck spends at each station has increased to 0.72 hours at station A, 0.48 hours at station B, and 0.6 hours at station C.

*Use the following information to answer the next question.*

The owner of a small amusement park represents the number and type of vehicles that are in his parking lot on a particular Thursday, Friday, and Saturday using the matrix below.

**Matrix A:**

| Day of Week | Type of Vehicle: Car | Bus |
|---|---|---|
| T | 85 | 12 |
| F | 43 | 17 |
| S | 102 | 33 |

He makes a second matrix to indicate the parking cost of \$8 per car and \$22 per bus.

**Matrix B:**

| Type of Vehicle | Parking Cost |
|---|---|
| Car | 8 |
| Bus | 22 |

**Written Response—10%**

**8.** **a.** What does the value 33 in matrix A represent?

**SOLUTION to part a**

There are 33 buses in the parking lot on Saturday.

**b.** Use matrix multiplication to calculate the revenue for each of the three days. Write a statement that describes the result of this multiplication.

**A POSSIBLE SOLUTION to part b**

$$\begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 944 \\ 718 \\ 1542 \end{bmatrix}$$

The revenue for each day is \$944, \$718, and \$1 542 for Thursday, Friday, and Saturday, respectively.

**c.** Use matrix operations to calculate an increase of 10% in the daily parking price. Show all calculations.

**A POSSIBLE SOLUTION to part c**

$$1.10 \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 8.8 \\ 24.2 \end{bmatrix}$$

The new parking price is \$8.80 for cars and \$24.20 for buses.

**or**

$$0.1 \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} = \begin{bmatrix} 0.8 \\ 2.2 \end{bmatrix}$$

The parking prices will increase by \$0.80 for cars and \$2.20 for buses.

SE **d.** How much **more** money would the owner have made on Saturday as a result of a 10% price increase?

**A POSSIBLE SOLUTION to part d**

$$\begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8 \\ 22 \end{bmatrix} \times 1.10 = \begin{bmatrix} 85 & 12 \\ 43 & 17 \\ 102 & 33 \end{bmatrix} \times \begin{bmatrix} 8.8 \\ 24.2 \end{bmatrix}$$

$$= \begin{bmatrix} 1038.4 \\ 789.8 \\ 1696.2 \end{bmatrix}$$

**or**

$$1.1 \times \begin{bmatrix} 944 \\ 718 \\ 1542 \end{bmatrix} = \begin{bmatrix} 1038.4 \\ 789.8 \\ 1696.2 \end{bmatrix}$$

New revenue = \$1 696.20
Orig. revenue = \$1 542.00

\$1 696.20 – \$1 542 = \$154.20

The owner would have made \$154.20 more on Saturday.

*Use the following information to answer the next question.*

A soccer league collected the following statistics over eight games.

| | Win | Tie | Loss |
|---|---|---|---|
| Bulldogs | 4 | 3 | 1 |
| Titans | 7 | 1 | 0 |
| Rovers | 2 | 2 | 4 |

Each team earns 2 points for a win, 1 point for a tie, and 0 points for a loss.

**9.** Which of the following matrix operations could be used to determine the points earned by each team after eight games?

**A.** $\begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

**B.** $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

**C.** $\begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 1 & 0 \end{bmatrix}$

**D.** $\begin{bmatrix} 2 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

**Solution:**

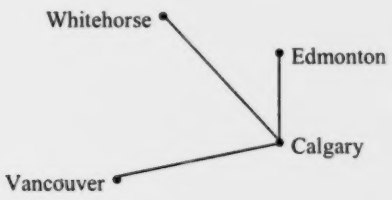
To get a $3 \times 1$ matrix that shows the total number of points earned by each team, the number of wins, ties and losses for each team $\begin{bmatrix} 4 & 3 & 1 \\ 7 & 1 & 0 \\ 2 & 2 & 4 \end{bmatrix}$ needs to be multiplied by the points awarded for each $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

In order to perform matrix multiplication, the number of columns in the first matrix must match the number of rows in the second matrix. Therefore the correct response is C.

**10.** A particular company's air freight is transported among four cities in Western Canada. Calgary is the hub of the network, which means that all air freight either originates in Calgary or is transported through Calgary.

**a.** Set up a network matrix, A, for the cities given below. Use 1 to represent that there is a direct freight transport between two cities and 0 to indicate that there is no direct freight transport between two cities.

**Cities in Network A**




**b.** If matrix A is squared, matrix $A^2$ can represent the network matrix for one-way routes and round-trip routes that have exactly one stopover. A stopover occurs when the plane stops in a city enroute to its final destination. Evaluate matrix $A^2$.

**SE** **c.** Explain why there is one element equal to 3, nine elements equal to 1, and six elements equal to 0 in the matrix $A^2$.

**Solution:**

**a.** **Matrix A**

| | C | E | W | V |
|---|---|---|---|---|
| C | 0 | 1 | 1 | 1 |
| E | 1 | 0 | 0 | 0 |
| W | 1 | 0 | 0 | 0 |
| V | 1 | 0 | 0 | 0 |

Calgary is the hub of the network and Edmonton, Whitehorse, and Vancouver are all spokes.

**b.** **Matrix $A^2$**

| | C | E | W | V |
|---|---|---|---|---|
| C | 3 | 0 | 0 | 0 |
| E | 0 | 1 | 1 | 1 |
| W | 0 | 1 | 1 | 1 |
| V | 0 | 1 | 1 | 1 |

Each entry in $A^2$ represents the number of one-way routes and round-trip routes that have exactly one stopover.

**SE** **c.** There is only one element equal to 3 because there are 3 routes with exactly one stopover. You can fly to any one of the three spokes from the Calgary hub and back, with one stopover.

There are nine elements equal to 1 because you can fly from any of the spokes, Edmonton, Whitehorse, or Vancouver, and back with one stopover in Calgary (3 routes), or you can fly between any two of the cities with one stopover (6 routes). This gives a total of 9 routes.

There are six elements equal to 0 because there is no way to fly from the hub to any spoke with exactly one stopover.

**11.** The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Calgary as the hub.

$$\begin{array}{c} \\ C \\ E \\ V \end{array}\begin{array}{c} \begin{array}{ccc} C & E & V \end{array} \\ \begin{bmatrix} 0 & 300 & 800 \\ 300 & 0 & 0 \\ 800 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Calgary is the hub, then the sum of the distances from the hub to each of the spoke cities and back is (300 + 800 + 300 + 800) or 2 200 km.

**a.** The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Vancouver as the hub.

$$\begin{array}{c} \\ V \\ E \\ C \end{array}\begin{array}{c} \begin{array}{ccc} V & E & C \end{array} \\ \begin{bmatrix} 0 & 1000 & 800 \\ 1000 & 0 & 0 \\ 800 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Vancouver is the hub, then the sum of the distances from the hub to each of the spoke cities and back is _____.

**b.** The matrix below shows the distances of airline routes between Vancouver, Calgary, and Edmonton, with Edmonton as the hub.

$$\begin{array}{c} \\ E \\ V \\ C \end{array}\begin{array}{c} \begin{array}{ccc} E & V & C \end{array} \\ \begin{bmatrix} 0 & 1000 & 300 \\ 1000 & 0 & 0 \\ 300 & 0 & 0 \end{bmatrix} \end{array}$$

Given that Edmonton is the hub, then the sum of the distances from the hub to each of the spoke cities and back is _____.

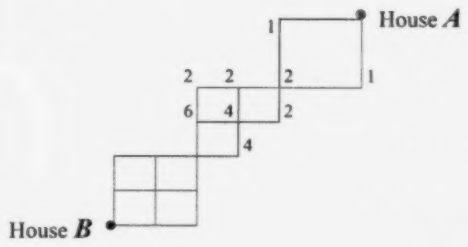
**c.** Which is the most efficient hub?

**Solution:**

**a.** (1 000 + 800 + 1 000 + 800) or 3 600 km
**b.** (1 000 + 300 + 1 000 + 300) or 2 600 km
**c.** Calgary is the most efficient hub, followed by Edmonton and then Vancouver.

*Use the following information to answer the next question.*

A paperboy who delivers papers on his bike can travel only on the trails represented in the diagram below.



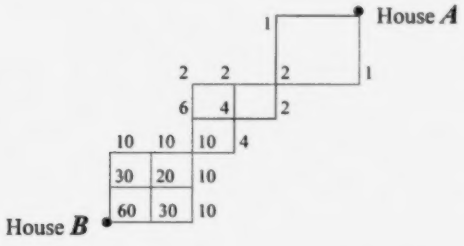

The number beside a vertex indicates the number of trails that lead to that vertex.

SE **12.** The number of different trails that the paperboy can take to get from house ***A*** to house ***B*** without backtracking is

**A.** 36
**B.** 60
**C.** 72
**D.** 120

**Solution:**

The diagram above can be completed in the following manner:




There are 60 different trails that the paperboy can take from house A to house B without backtracking.

*Use the following information to answer the next question.*

In 1998, a truck dealership sold 300 blue trucks, 100 green trucks, and 200 red trucks. In 1999, sales of blue trucks decreased by 10%, sales of green trucks increased by 20%, and sales of red trucks increased by 30%. This situation can be modelled by the matrix operation shown below.

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 300 \\ 100 \\ 200 \end{bmatrix} = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix}$$

**Written Response—15%**

**13.** **a.** Calculate the product of the matrices above.

**A POSSIBLE SOLUTION to part a**

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 300 \\ 100 \\ 200 \end{bmatrix} = \begin{bmatrix} 270 \\ 120 \\ 260 \end{bmatrix}$$

**b.** How many green trucks were sold in 1999?

**A POSSIBLE SOLUTION to part b**

$100 \times 1.2 = 120$ green trucks sold.

c. Assume that the pattern continues. Use matrix multiplication to determine the sales for each colour of truck in the year 2000.

**A POSSIBLE SOLUTION to part c**

$$\begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.3 \end{bmatrix} \times \begin{bmatrix} 270 \\ 120 \\ 260 \end{bmatrix} = \begin{bmatrix} 243 \\ 144 \\ 338 \end{bmatrix}$$

Sales of blue trucks will be 243, green trucks will be 144, and red trucks will be 338.

*Use the following additional information to answer the next part of the question.*

In 1998, of all vehicles sold at a different vehicle dealership, 800 were trucks and 600 were sport utility vehicles (SUVs).

In 1999, 8% of the truck owners traded in their trucks for new SUVs, and 22% of the SUV owners traded in their SUVs for new trucks.

No SUV or truck owner switched to a different type of vehicle and no owner of other types of vehicles switched to an SUV or a truck.

SE

d. Represent this information by completing the $2 \times 2$ matrix below, and perform the matrix multiplication.

$$\begin{bmatrix} 800 & 600 \end{bmatrix} \times \begin{bmatrix} \underline{\quad} & 0.08 \\ 0.22 & \underline{\quad} \end{bmatrix}$$

**A POSSIBLE SOLUTION to part d**

$$\begin{bmatrix} 800 & 600 \end{bmatrix} \times \begin{bmatrix} \underline{0.92} & 0.08 \\ 0.22 & \underline{0.78} \end{bmatrix} = \begin{bmatrix} 868 & 532 \end{bmatrix}$$

**or**

$$\begin{bmatrix} 800 & 600 \end{bmatrix} \times \begin{bmatrix} \underline{0} & 0.08 \\ 0.22 & \underline{0} \end{bmatrix} = \begin{bmatrix} 132 & 64 \end{bmatrix}$$

- Explain what the product of the matrix multiplication in the bullet above means in this context.

| **A POSSIBLE SOLUTION to part d, bullet 1** |
| --- |
| In 1999, 868 people owned trucks bought from the dealership, and 532 owned SUVs bought from the dealership.<br>**or**<br>There were 132 people who traded their SUVs for trucks and 64 people who traded their trucks for SUVs. |

SE **14.** What conditions must be met in order to perform matrix multiplication? Justify your answer.

**Solution:**

To perform matrix multiplication, the number of columns in the first matrix must match the number of rows in the second matrix.
For example:

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is a 2 by 3 matrix. (2 rows and 3 columns)

and

$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ is a 3 by 3 matrix. (3 rows and 3 columns)

When A and B are multiplied, the procedure is as follows:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$

$$\begin{bmatrix} 1(1) + 2(4) + 3(7) & 1(2) + 2(5) + 3(8) & 1(3) + 2(6) + 3(9) \\ 4(1) + 5(4) + 6(7) & 4(2) + 5(5) + 6(8) & 4(3) + 5(6) + 6(9) \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 36 & 42 \\ 66 & 81 & 96 \end{bmatrix}$$

It is impossible to multiply two matrices in which the dimensions do not match, so a product cannot be found.
For example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = [1(1) + 2(3) + 3(\underline{\ ?\ })]$$

The number of columns in the first matrix does not equal the number of rows in the second matrix. Since there is a dimensional mismatch, a product cannot be found.

## Finance and Spreadsheets

### *General Outcome*

Design or use a spreadsheet to make and justify financial decisions.

**General Notes:**
- Teachers may wish to check with the computer system administrator about log ons for students before beginning this unit.
- This unit is intended to focus on the analysis of financial situations using a spreadsheet, but is not intended to be an intense, in-depth study of spreadsheets.
- Students are not expected to master specialized computer functions such as *If-Then-Else* statements.
- Teachers should be aware of time constraints during this unit as some analyses can become quite lengthy. The resource package provides templates of spreadsheets for analytical use.
- Teachers should spend approximately one quarter of the time devoted to this unit on design of the spreadsheet and the remaining time on analyses.
- Students should be familiar with absolute cell references.

### *Specific Outcomes*

**Specific Outcome 3.1**

Design a financial spreadsheet template to allow users to input their own variables. [C, PS, T]

**3.1 Note:**
- This outcome is intended to be achieved using computer spreadsheet technology rather than functions on some graphing calculators.

*Refer to examples 1, 8, and 10.*

**Specific Outcome 3.2**

Analyze the costs and benefits of renting or buying an increasing asset, such as a home. [C, CN, PS, T]

**3.2 Notes:**

- Communicating the results of the analysis is a key idea behind specific outcome 3.2.
- Although calculating mortgage or loan payments using the finance applications of a graphing calculator may be beneficial to classroom discussion, it will not be assessed on the diploma examination. Not all calculators on the approved list have this financial application function.
- Diploma examination questions relating to this outcome will focus on situations where the mortgage payment is known or where it can be calculated using a given table or chart.
- The graphing calculator assumes a constant interest rate and payment throughout the amortization of the loan. Interesting discussions may result when changes in rate and payment are considered.
- Much of the analysis can be done using an exponential regression model.
- Students are not required to calculate effective interest rates using a formula.

**Specific Outcome 3.3**

Analyze the costs and benefits of leasing or buying a decreasing asset, such as a vehicle or a computer. [C, CN, PS, T]

**3.3 Notes:**

- Communicating the results of the analysis is a key idea behind specific outcome 3.3.
- Limit spreadsheet construction (as this is not the focus of this outcome) to examples where the compounding period matches the payment period.
- The graphing calculator assumes a constant interest rate and payment throughout the amortization of the loan. Interesting discussions may result when changes in rate and payment are considered.
- Much of the analysis can be done using an exponential regression model.

*Refer to examples 4, 5, 6, and 7.*

**Specific Outcome 3.4**

Analyze an investment portfolio, applying such concepts as interest rate, rate of return and total return. [C, CN, PS, T]

**3.4 Note:**

- Students should be able to use the compound interest formula and recognize it as an exponential function of the form $y = a \cdot b^x$ with a $b$ value greater than 1.

*Refer to examples 2, 3, and 9.*

| *Acceptable Standard* | *Standard of Excellence* |
|---|---|
| The student can | The student can also |
| • distinguish the difference between entered values, fixed values, and computed values in a spreadsheet | |
| • identify appropriate algebraic functions used in the solution of spreadsheet problems | • interpret a written problem and develop an appropriate spreadsheet |
| • work within the parameters of an existing spreadsheet to solve non-recursive or recursive problems | • modify an existing spreadsheet to accommodate changing needs, and analyze differing factors in each model |
| • design a simple spreadsheet by using formulas and functions to solve non-recursive problems, such as billing and design calculations, or recursive problems such as loan calculations | • design more complex spreadsheets |
| • given a spreadsheet, determine the algebraic formulas used to construct the spreadsheet | |
| • given two schedules, correctly identify the more appropriate payment model and give appropriate justification for the choice in terms of cost | • set up a spreadsheet that can help in cost-and-benefit analysis |
| • offer a few pros and cons with respect to the chosen payment model | • offer pros and cons that show thorough understanding with respect to the chosen payment model |
| • use correct terminology to discuss payment options | |
| • determine whether renting or buying is a more appropriate choice, given specific circumstances | |
| • discuss the appropriateness of a regression model to a given context | |
| • recognize the value of an asset as either increasing or decreasing | |
| • recognize that the value of an increasing asset can be represented by the regression model $y = a \cdot b^x$, where $b > 1$ | • calculate average annual percentage appreciation by using exponential regression |
| • recognize that the value of a decreasing asset can be represented by the regression model $y = a \cdot b^x$, where $0 < b < 1$ | • calculate average annual percentage depreciation by using exponential regression |
| • use an exponential regression model to make predictions | |
| • use correct terminology to describe an investment portfolio | • compare two similar portfolios and make comments with regard to interest rate, rate of return, and total return |
| • calculate the total return and the average annual rate of return of an investment | |
| • participate in and contribute toward the problem-solving process for problems that require the analysis of finance and spreadsheets studied in Applied Mathematics 30 | • complete the solution to problems that require the analysis of finance and spreadsheets studied in Applied Mathematics 30 |

## *Examples*

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the *standard of excellence*.

**1.** For the following invoice, develop a spreadsheet that calculates the parts and labour costs and requires the operator to input a minimum number of entries.

**Acme Auto Parts**

**Customer Inquiries**

| Item No. | Auto Parts | Quantity | Unit Price | Total | Labour | |
|---|---|---|---|---|---|---|
| 1 | Brake Pads | 1 | $26.34 | $26.34 | OH/Front Brakes | $47.70 |
| | | | | | (0.9 hrs. @ $53.00/hr.) | |
| 2 | Wheel Seals | 2 | $5.25 | $10.50 | Machined and | $10.00 |
| | | | | | Replaced Rotor | |
| | | | | | (flat rate) | |
| 3 | Rotor | 1 | $30.16 | $30.16 | | |
| | | | | | | |
| | | | Total Parts | $67.00 | Total Labour | $57.70 |
| | | | | | Subtotal | $124.70 |
| | | | | | GST (7%) | $8.73 |
| | | | | | Total | $133.43 |

**Solution:**

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | | | **Acme Auto Parts** | | | |
| 2 | **Customer Inquiries** | | | | | | |
| 3 | Item No. | Auto Parts | Quantity | Unit Price | | Labour | |
| 4 | 1 | Brake Pads | 1 | $26.34 | [illegible] | OH/Front Brakes | $47.70 |
| 5 | | | | | | (0.9 hrs. @ $53.00/hr.) | |
| 6 | 2 | Wheel Seals | 2 | $5.25 | [illegible] | Machined and | $10.00 |
| 7 | | | | | | Replaced Rotor | |
| 8 | | | | | | (flat rate) | |
| 9 | 3 | Rotor | 1 | $30.16 | [illegible] | | |
| 10 | | | | | | | |
| 11 | | | | Total Parts | [illegible]0) | Total Labour | [illegible]0) |
| 12 | | | | | | Subtotal | [illegible]61 |
| 13 | | | | | | GST (7%) | [illegible]0.07 |
| 14 | | | | | | Total | [illegible]63 |

*Use the following information to answer the next two questions.*

Owen invested $2 000 three years ago. During this time, he has been tracking the investment by recording its value at the end of each year.

| Year | Value |
|---|---|
| 0 | $2 000.00 |
| 1 | $2 160.00 |
| 2 | $2 309.04 |
| 3 | $2 477.98 |

**2.** The average annual rate of return on this investment, to the nearest tenth of a percentage, is

**A.** 0.7%/a

**B.** 1.1%/a

**C.** 1.2%/a

**D.** 7.4%/a

**Solution:**

**Method 1**

Enter values and perform an exponential regression:

$$y = 2004.59....(1.0735....)^x$$

In this expression, $y = a \cdot b^x$, the value of $b$ is 1.0735..... This value indicates that the investment is increasing at an average annual rate of 7.35...% (7.4%).

**Method 2**

| Year | Percentage Growth |
|---|---|
| 0–1 | 8% |
| 1–2 | 6.9% |
| 2–3 | 7.3% |

Average Percentage Growth

$$\frac{8+6.9+7.3}{3} = 7.4\%$$

3. The total **return** on this investment after year 3 is

   **A.** $168.94
   **B.** $477.98
   **C.** $6 947.02
   **D.** $8 947.02

**Solution:**

At the end of year 3, the $2 000 investment is worth $2 477.98. Therefore, the total return is $477.98.

*Use the following information to answer the next question.*

John is considering whether to buy or lease a car. The price of the car he is interested in is $25 500. Regardless of the option he chooses, John will make a down payment of $3 000.

John's first option is to purchase the car. He will have to make payments of $557.25/mo for 48 mo.

John's second option is to lease the car. He will have to make payments of $307.50/mo for 48 mo, after which time he will still owe $12 850.

4. *If John wants to own the car at the end of 48 months, he should* ___*i*___ *the car because he will save $*___*ii*___.

The row that completes this statement is row

| Row | *i* | *ii* |
|---|---|---|
| **A.** | lease | $862 |
| **B.** | buy | $862 |
| **C.** | lease | $11 988 |
| **D.** | buy | $11 988 |

**Solution:**

Purchase: $557.25 × 48 = $26 748
Lease: $307.50 × 48 = $27 610

If John wants to own the car at the end of 48 months, he should buy the car because he will save $862.

*Use the following information to answer the next question.*

It is predicted that the value of John's car will depreciate every year for the first four years according to the table below.

| Year | Value |
|---|---|
| 0 | $25 500 |
| 1 | $17 850 |
| 2 | $16 065 |
| 3 | $14 459 |
| 4 | $13 012 |

SE **5.** If John uses an exponential regression equation to determine the value of his car in the future, then he will find that the average rate of **depreciation** is

**A.** 14.4% per year

**B.** 49.0% per year

**C.** 51.0% per year

**D.** 85.6% per year

**Solution:**

The data can be modelled by the exponential function $y = 23\ 061.45....(0.8558...)^x$. The average annual rate of depreciation is $1 - 0.8558... = 0.14412...$ (14.4%).

**6.** Chris is looking into the purchase/lease of a car. The vehicle he has chosen is priced at $21 720.00 plus GST, and financing is calculated at 7.5%/a, compounded monthly. Chris will make a down payment of $2 000 and has the following two options available.

**Option 1: (Lease with buyout)**

| | |
|---|---|
| Amount financed | $19 720 |
| Monthly lease payments | $377.55 + GST on monthly payment + $\frac{\text{GST on down payment}}{36}$ = 407.87 |
| Number of months | 36 |
| Final buyout | $9 394.06 + GST = $10 051. 64 |

**Option 2: (Finance full amount)**

| | |
|---|---|
| Purchase price | $21 720 + GST = $23 240.40 |
| Amount financed | $23 240.40 – 2000.00 = $21 240.40 |
| Monthly payment | $660.71 |
| Number of months | 36 |

**Note**: When leasing a car, the GST is calculated on the monthly payment. The GST on the down payment is divided by the number of months and also added to the monthly payment. Lease payments must be made at the beginning of the period.

**a.** How much does Chris spend in total for monthly payments over the term of the financing for each option? Explain why is there a difference between the two amounts. Explain.

**b.** What is the total amount spent by Chris in each option if he would like to own the vehicle at the end of 36 months? What is the difference in final price? Explain.

**c.** Which option would you consider to be best for Chris and in which circumstances? Explain.

**SE** **d.** How much would Chris have to save each month at 6%/a, compounded monthly, to pay for the $10 051.64 buyout at the end of 36 months? Would this decrease his final cost for the lease or not? Explain.

**Solution:**

**a.** **Option 1** **Option 2**
$407.87 \times 36 = 14\,683.32$ $660.71 \times 36 = 23\,785.56$
The difference is that in option 2, Chris owns the car, and in option 1, Chris does not own the car.

**b.** The total cost for option 1 would be \$26 734.96, which includes all payments, the down payment of \$2 000, and the final buyout of \$10 051.64. The total amount spent by Chris in option 2 would be \$25 785.56, which includes all payments plus the down payment of \$2 000. The difference in final price to purchase the car would be \$949.40. It is assumed that Chris has the \$10 051.64 for the final buyout and does not have to finance this portion as well.

**c.** If Chris can afford the monthly payment of \$660.71 for 36 months, he will save \$949.40. If Chris cannot afford the higher payment, then the lease value is better, but he must remember to save the money for the buyout over the next 36 months.

SE **d.** To save the buyout amount over 36 months at 6%/a, compounded monthly, Chris would have to save \$255.53 per month or \$9 199.08; this would amount to \$10 051.64, including interest. This would cut his final cost to \$25 882.40, which includes \$2 000 down payment, \$14 683.32 in lease payments, and \$9 199.08 in savings for 36 months.

**Note:** The solution to part d requires the use of the finance application of a graphing calculator.

7. A person borrows $15 000 to purchase a new car. The loan is for 36 months at 8%/a, compounded monthly, with payments of $470.05/mo. Construct a spreadsheet that will allow the person to check the amount owing at any time during the three years.

**Solution:**

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | Loan Information | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | Period of Loan (months) | 36 | | | | |
| 6 | Amount Borrowed | 15 000 | | | | |
| 7 | Monthly Payment | 470.05 | | | | |
| 8 | Interest Rate | 0.08 | | | | |
| 9 | | | | | | |
| 10 | | | | Balance | | New |
| 11 | | Month | Interest | with Interest | Payment | Balance |
| 12 | | 1 | =B6*($B$8/12) | =B6+C12 | =$B$7 | =D12-E12 |
| 13 | | =B12+1 | =F12*($B$8/12) | =F12+C13 | =$B$7 | =D13-E13 |
| 14 | | | | | | |
| 15 | | | | | | |
| 16 | | | | | | |
| 17 | | | | | | |
| 18 | | | | | | |
| 19 | | | | | | |
| 20 | | | | | | |
| 21 | | | | | | |
| 22 | | | | | | |
| 23 | | | | | | |
| 24 | | | | | | |
| 25 | | | | | | |
| 26 | | | ***Use the "fill down" command to enter the formulas*** | | | |
| 27 | | | ***for the remainder of the spreadsheet.*** | | | |
| 28 | | | | | | |
| 29 | | | | | | |
| 30 | | | | | | |
| 31 | | | | | | |
| 32 | | | | | | |
| 33 | | | | | | |
| 34 | | | | | | |
| 35 | | | | | | |
| 36 | | | | | | |
| 37 | | | | | | |
| 38 | | | | | | |
| 39 | | | | | | |
| 40 | | | | | | |
| 41 | | | | | | |
| 42 | | | | | | |
| 43 | | | | | | |
| 44 | | | | | | |
| 45 | | | | | | |
| 46 | | | | | | |
| 47 | | =B46+1 | =F46*($B$8/12) | =F46+C47 | =$B$7 | =D47-E47 |
| 48 | | | | | | |
| 49 | **Totals** | | =SUM(C12:C47) | | =SUM(E12:E47) | |
| 50 | | | | | | |

The previous spreadsheet will result in the following values:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | | | | |
| 2 | | | Loan Information | | | |
| 3 | | | | | | |
| 4 | | | | | | |
| 5 | Period of Loan (months) | 36 | | | | |
| 6 | Amount Borrowed | $15 000.00 | | | | |
| 7 | Monthly Payment | $470.05 | | | | |
| 8 | Interest Rate | 8.00% | | | | |
| 9 | | | | | | |
| 10 | | | | Balance | | New |
| 11 | | Month | Interest | with Interest | Payment | Balance |
| 12 | | 1 | $100.00 | $15 100.00 | $470.05 | $14 629.95 |
| 13 | | 2 | $97.53 | $14 727.48 | $470.05 | $14 257.43 |
| 14 | | 3 | $95.05 | $14 352.48 | $470.05 | $13 882.43 |
| 15 | | 4 | $92.55 | $13 974.98 | $470.05 | $13 504.93 |
| 16 | | 5 | $90.03 | $13 594.96 | $470.05 | $13 124.91 |
| 17 | | 6 | $87.50 | $13 212.41 | $470.05 | $12 742.36 |
| 18 | | 7 | $84.95 | $12 827.31 | $470.05 | $12 357.26 |
| 19 | | 8 | $82.38 | $12 439.65 | $470.05 | $11 969.60 |
| 20 | | 9 | $79.80 | $12 049.39 | $470.05 | $11 579.34 |
| 21 | | 10 | $77.20 | $11 656.54 | $470.05 | $11 186.49 |
| 22 | | 11 | $74.58 | $11 261.06 | $470.05 | $10 791.01 |
| 23 | | 12 | $71.94 | $10 862.95 | $470.05 | $10 392.90 |
| 24 | | 13 | $69.29 | $10 462.19 | $470.05 | $9 992.14 |
| 25 | | 14 | $66.61 | $10 058.76 | $470.05 | $9 588.71 |
| 26 | | 15 | $63.92 | $9 652.63 | $470.05 | $9 182.58 |
| 27 | | 16 | $61.22 | $9 243.80 | $470.05 | $8 773.75 |
| 28 | | 17 | $58.49 | $8 832.24 | $470.05 | $8 362.19 |
| 29 | | 18 | $55.75 | $8 417.94 | $470.05 | $7 947.89 |
| 30 | | 19 | $52.99 | $8 000.87 | $470.05 | $7 530.82 |
| 31 | | 20 | $50.21 | $7 581.03 | $470.05 | $7 110.98 |
| 32 | | 21 | $47.41 | $7 158.38 | $470.05 | $6 688.33 |
| 33 | | 22 | $44.59 | $6 732.92 | $470.05 | $6 262.87 |
| 34 | | 23 | $41.75 | $6 304.63 | $470.05 | $5 834.58 |
| 35 | | 24 | $38.90 | $5 873.47 | $470.05 | $5 403.42 |
| 36 | | 25 | $36.02 | $5 439.45 | $470.05 | $4 969.40 |
| 37 | | 26 | $33.13 | $5 002.53 | $470.05 | $4 532.48 |
| 38 | | 27 | $30.22 | $4 562.69 | $470.05 | $4 092.64 |
| 39 | | 28 | $27.28 | $4 119.93 | $470.05 | $3 649.88 |
| 40 | | 29 | $24.33 | $3 674.21 | $470.05 | $3 204.16 |
| 41 | | 30 | $21.36 | $3 225.52 | $470.05 | $2 755.47 |
| 42 | | 31 | $18.37 | $2 773.84 | $470.05 | $2 303.79 |
| 43 | | 32 | $15.36 | $2 319.15 | $470.05 | $1 849.10 |
| 44 | | 33 | $12.33 | $1 861.43 | $470.05 | $1 391.38 |
| 45 | | 34 | $9.28 | $1 400.65 | $470.05 | $930.60 |
| 46 | | 35 | $6.20 | $936.81 | $470.05 | $466.76 |
| 47 | | 36 | $3.11 | $469.87 | $470.05 | -$0.18 |
| 48 | | | | | | |
| 49 | Totals | | $1 921.62 | | $16 921.80 | |
| 50 | | | | | | |

*Use the following information to answer the next question.*

The following spreadsheet shows the beginning of an amortization table for 5 equal monthly payments to be made on a $900 loan with an interest rate of 1% per month.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **Payment Number** | **Payment** | **Interest Payment** | **Payment to Principal** | **Balance Remaining** |
| 2 | 1 | $155.29 | $9.00 | $146.29 | $753.71 |
| 3 | 2 | $155.29 | $7.54 | $147.75 | $605.96 |
| 4 | 3 | $155.29 | | | |
| 5 | 4 | | | | |
| 6 | 5 | | | | |

**8.** Which of the following formulas can be used to calculate the value of cell D4?

**A.** =B4 + C4

**B.** =E3 – B4

**C.** =B4 – 0.1*E3

**D.** =B4 – 0.01*E3

**Solution:**

Payment to principal is determined by subtracting the interest paid (1%) from the monthly payment. Therefore, a formula that will work is =B4 – 0.01 *E3.

*Use the following information to answer the next question.*

A manufacturer has two plants: at one, acoustic guitars are made, at the other, electric guitars are made. The following table shows some information related to each plant.

| | Acoustic Guitar Plant | | Electric Guitar Plant | |
|---|---|---|---|---|
| | **Model I** | **Model II** | **Model I** | **Model II** |
| **Material per guitar** | $ 60 | $ 128 | $ 35 | $ 93 |
| **Labour per guitar** | $145 | $ 275 | $210 | $290 |
| **Selling price per guitar** | $500 | $1 500 | $510 | $750 |
| **Number made and sold per month** | 120 | 75 | 251 | 109 |

**Written Response—10%**

**9.** **a.** Determine the total monthly revenue for each plant.

**A POSSIBLE SOLUTION to part a**

Acoustic Guitar Plant:
Revenue = 120 ($500) + 75 ($1 500)
= $172 500.00

Electric Guitar Plant:
Revenue = 251 ($510) + 109 ($750)
= $209 760.00

*Use the following additional information to answer the next part of the question.*

In addition to the cost for materials and labour, the company must cover operating costs.

The operating costs are $40 000/mo for the acoustic guitar plant and $28 000/mo for the electric guitar plant.

**b.** Determine the monthly profit for each plant.

**A POSSIBLE SOLUTION to part b**

AGP: $172\,500 - 120\,(60+145) - 75(128+275) - 40\,000 = \$77\,675.00$

EGP: $209\,760 - 251\,(35+210) - 109(93+290) - 28\,000 = \$78\,518.00$

The monthly profit for the acoustic guitar plant is $77 675.00 and $78 518.00 for the electric guitar plant.

**c.** Calculate the monthly profit
- per acoustic guitar
- per electric guitar

**A POSSIBLE SOLUTION to part c**

Acoustic guitar: $\frac{77\,675}{195} = \$398.33$

Electric Guitar: $\frac{78\,518}{360} = \$218.11$

The company profits \$398.33/mo on each acoustic guitar, and \$218.11/mo on each electric guitar.

**d.** The company is experiencing financial difficulty and wishes to close one plant. Which plant should be closed? Explain your answer.

**A POSSIBLE SOLUTION to part d**

Each of the following answers is sufficient

- The company should close the acoustic guitar plant because it makes the lowest monthly profit.

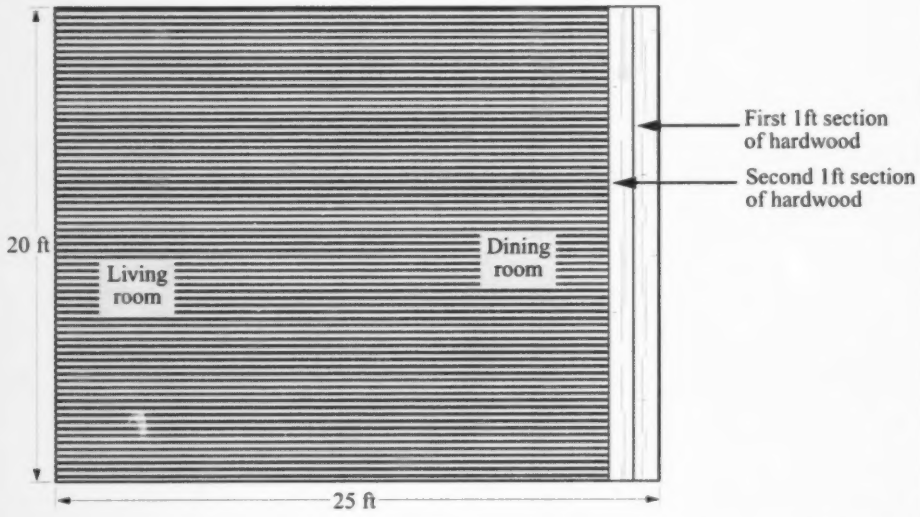
**or**

- The company should close the electric guitar plant because it makes the lowest profit per guitar.

*Use the following information to answer the next question.*

A family is planning to cover their $20\text{ ft} \times 25\text{ ft}$ living room/dining room with carpet and hardwood. Their budget for this project is \$5 000. The cost of the carpet is $\$7.25/\text{ft}^2$, and the cost of the hardwood is $\$12.50/\text{ft}^2$.

The family would like to cover the entire area with hardwood, but they know that it would cost more than the budgeted amount. In order to determine the maximum area of hardwood that they can afford, they calculated the total cost of the flooring each time a 1 ft wide section of hardwood was added to the room, as shown below.



The family used the following spreadsheet to calculate costs and to determine the maximum area of hardwood that they could afford.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **Area of floor with hardwood ($ft^2$)** | **Area of floor with carpet ($ft^2$)** | **Cost of hardwood @ \$12.50/$ft^2$** | **Cost of carpet @ \$7.25/$ft^2$** | **Total cost of flooring** |
| 2 | 500 | 0 | \$6 250 | \$0 | \$6 250 |
| 3 | 480 | 20 | \$6 000 | \$145 | \$6 145 |
| 4 | 460 | 40 | \$5 750 | \$290 | \$6 040 |
| 5 | 440 | 60 | \$5 500 | \$435 | \$5 935 |
| 6 | 420 | 80 | \$5 250 | \$580 | \$5 830 |
| 7 | 400 | 100 | \$5 000 | \$725 | \$5 725 |
| 8 | 380 | 120 | \$4 750 | \$870 | \$5 620 |
| 9 | 360 | 140 | \$4 500 | \$1 015 | \$5 515 |
| 10 | 340 | 160 | \$4 250 | \$1 160 | \$5 410 |
| 11 | 320 | 180 | \$4 000 | \$1 305 | \$5 305 |
| 12 | 300 | 200 | \$3 750 | \$1 450 | \$5 200 |
| 13 | 280 | 220 | \$3 500 | \$1 595 | \$5 095 |
| 14 | 260 | 240 | \$3 250 | \$1 740 | \$4 990 |
| 15 | 240 | 260 | \$3 000 | \$1 885 | \$4 885 |
| 16 | 220 | 280 | \$2 750 | \$2 030 | \$4 780 |
| 17 | 200 | 300 | \$2 500 | \$2 175 | \$4 675 |
| 18 | 180 | 320 | \$2 250 | \$2 320 | \$4 570 |
| 19 | 160 | 340 | \$2 000 | \$2 465 | \$4 465 |
| 20 | 140 | 360 | \$1 750 | \$2 610 | \$4 360 |
| 21 | 120 | 380 | \$1 500 | \$2 755 | \$4 255 |
| 22 | 100 | 400 | \$1 250 | \$2 900 | \$4 150 |
| 23 | 80 | 420 | \$1 000 | \$3 045 | \$4 045 |
| 24 | 60 | 440 | \$750 | \$3 190 | \$3 940 |
| 25 | 40 | 460 | \$500 | \$3 335 | \$3 835 |
| 26 | 20 | 480 | \$250 | \$3 480 | \$3 730 |
| 27 | 0 | 500 | 0 | \$3 625 | \$3 625 |

**Written Response—10%**

**10.** **a.** Explain the relationship between the values in columns A and B.

**A POSSIBLE SOLUTION to part a**

The sum of column A and the sum of column B each equal 500 $ft^2$ (the total floor area of the living room and dining room).

**b.** Show, by writing a statement or a formula, how the value in cell E9 ($5 515) was calculated. Make reference to other cells in row 9.

**A POSSIBLE SOLUTION to part b**

Cell C9 is calculated by multiplying A9, the total area of hardwood, by $12.50. Cell D9 is calculated by multiplying B9, the total area of carpet, by $7.25. Cell E9 is then calculated by adding cells C9 and D9 to find the total cost of flooring for the given areas of carpet and hardwood.

**or**

E9 = A9 * 12.50 + B9 * 7.25

**or**

C9 = A9 * 12.50
D9 = B9 * 7.25
C9 + D9 = E9

**or**

C9 + D9 = E9

**c.** If the family is to remain within their budget, what is the maximum area of hardwood that they can place into this living room/dining room area?

**A POSSIBLE SOLUTION to part c**

To remain within budget, the maximum area of hardwood is a 13 ft × 20 ft piece or 260 $ft^2$.

**d.** • What is the total cost for this plan?

| **POSSIBLE SOLUTIONS to part d, bullet 1** |
|---|
| **Option 1:**<br>The family could have 250 $ft^2$ each of carpet and hardwood for a total cost of $4 937.50.<br><br>**Option 2:** reference row 14<br>260 $ft^2$ of hardwood and 240 $ft^2$ of carpet for a total cost of $4 990.<br><br>**Option 3:** reference row 15<br>240 $ft^2$ of hardwood and 260 $ft^2$ of carpet for a total cost of $4 885. |

• Will the family remain within their budget? Explain.

| **A POSSIBLE SOLUTION to part d, bullet 2** |
|---|
| The total cost is less than $5 000, so the family will remain within their budget. |

## Cyclic, Recursive, and Fractal Patterns

### *General Outcome*

Generate and analyze cyclic, recursive and fractal patterns.

**General Notes:**

- It is not the intent of this unit to study sinusoidal curves and characteristics of fractal patterns, but to use these, and other functions, as regression models to show patterns from which inferences can be drawn.
- These outcomes require substitution of calculator-generated parameters into the regression equations and an understanding of the significance of these values.
- A fractal is a geometric figure exhibiting self-similarity and is produced by repeated iterations. Each successive fractal in a fractal pattern is more complex than the previous one.

### *Specific Outcomes*

**Specific Outcome 4.1**

Collect sinusoidal data, graph the data using technology, and represent the data with a best fit equation of the form:

- $y = a\sin(bx + c) + d$. [C, CN, PS, T, V]

**4.1 Notes:**

When using technology to generate a sinusoidal regression model, note the following points.

- Graphing calculators will model sinusoidal regression in the form $y = a \cdot \sin(bx + c) + d$, where $x$ is measured in radians. This means that the period of the function is $\frac{2\pi}{b}$ units and the horizontal transformation is to the left or right.

  To graph a sinusoidal regression model, students' calculators must be in radian mode. This means that teachers will have to explain to students that there are two ways to express angle measures: in radians and in degrees. It is not the intent to teach conversions from one measure to another, but simply to make students aware of the two methods.
- A minimum of five points over most of one period is normally sufficient to produce an adequate regression model. Creating a list of eight or more points over two periods produces a better model. It is recommended that when entering values for sinusoidal regression into a graphing calculator, students begin with the lowest $x$ value.
- Sinusoidal regression models on a graphing calculator will occasionally have very small values for one or more of the parameters. Students will require practice recognizing these values and should write values less than or equal to $10^{-4}$ as zero.
- Recognize that a value of $c$ other than zero is an indication that the initial point of the scenario is to the left or right of the *y-axis*.

- Determining the exact value of the horizontal transformation $\left(\frac{c}{b}\right)$ is beyond the scope of Applied Mathematics 30.
- The notation for a sinusoidal regression equation in Applied Mathematics 30 is $y = a \sin (bx + c) + d$, and in Pure Mathematics 30 it is $y = a \sin b(x \pm c) + d$.

*Refer to example 1.*

**Specific Outcome 4.2**

Use best-fit sinusoidal equations and their associated graphs to make predictions (interpolation, extrapolation). [C, CN, PS, T]

**4.2 Note:**
- Students should be able to make predictions from equations and graphs.

*Refer to examples 5, 6, and 7.*

**Specific Outcome 4.3**

Describe periodic events, including those represented by sinusoidal curves, using the terms amplitude, period, maximum and minimum values, and vertical and horizontal shift, and relating these terms to the parameters $a$, $b$, $c$, and d described in 4.1. [C, V]

**4.3 Notes:**
- Graphs should be provided for questions relating to non-sinusoidal periodic events.
- Descriptions of sinusoidal events can be based on a context, a graph, or an equation.

*Refer to examples 3 and 4.*

**Specific Outcome 4.4**

Use technology to generate and graph sequences that model real-life phenomena. [PS, T, V]

**4.4 Notes:**
- Students should be given an opportunity to determine an equation, without the use of technology, that models a sequence.
- Students should be familiar with regression models from Applied Mathematics 10 and 20 and should be able to determine which regression is the "best fit" for a particular set of data.
- Regression models are sufficient for this outcome. Students are not required to be familiar with the SEQ mode on some graphing calculators.

*Refer to examples 2, 8, 9, 10, and 11.*

**Specific Outcome 4.5**

Use technology to construct a fractal pattern by repeatedly applying a procedure to a geometric figure. [CN, R, V, T]

**4.5 Notes:**

- The use of technology in this outcome is intended to allow students to test conjectures without intensive labour. Students should, however, be able to generate the first few fractal patterns from simple figures such as regular polygons. Grid paper will help students create accurate fractal shapes.
- Dynamic software may be appropriate for classwork but assessment should be with paper and pencil only. Formulas can be generated by regression models.
- Students should be familiar with regression models from Applied Mathematics 10 and 20 and should be able to determine which regression is the "best fit" for a particular set of data.
- Define iterations as repeated recursive calculations or procedures.

*Refer to examples 12 and 16.*

**Specific Outcome 4.6**

Use the concept of self-similarity to compare and/or predict the perimeters, areas, and volumes of fractal patterns. [CN, R, T, V]

**4.6 Notes:**

- Be aware that fractal patterns can become very difficult very quickly. Look for examples that can be modelled by exponential regression. This can usually be accomplished by focusing on the number of new shapes at each iteration rather than on the total number of shapes.
- Two important properties of fractal patterns are
  - self-similarity (characteristics of the shape are maintained under any magnification)
  - increasing complexity of the shape at each iteration
- For area and volume fractal patterns, the pattern does not establish itself between the original and the first iteration.
- The volumes resulting from successive iterations of three-dimensional fractal patterns do not form fractals. This fact may lead to interesting discussions if limited to fairly simple shapes with reasonable numbers.
- Exponential regression cannot be performed on fractal patterns involving increasing areas and volumes.

*Refer to examples 13, 14, and 15.*

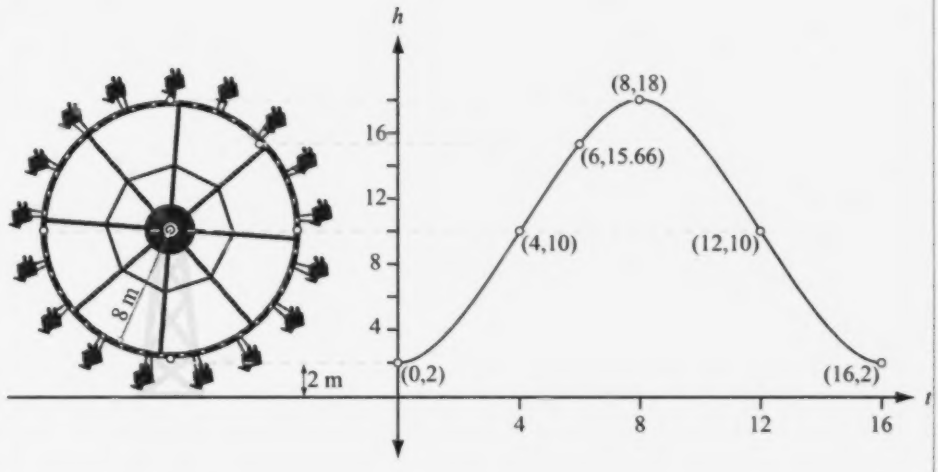
| *Acceptable Standard*<br>The student can | *Standard of Excellence*<br>The student can also |
|---|---|
| • use technology to graph data that represents a sinusoidal curve | |
| • state appropriate graphing calculator window settings as an ordered triple in the form $x$: [$x_{min}$, $x_{max}$, $x_{scale}$] and $y$: [$y_{min}$, $y_{max}$, $y_{scale}$] | |
| • use the regression model $y = a \sin(bx + c) + d$ to determine a best-fit equation | • provide explanations for trends in data |
| • use the TRACE or CALC function on a graphing calculator to make predictions | |
| • describe a periodic event, using correct terminology, given a diagram, a context, an equation, or a graph | • describe the transformational effect(s) of additional information on a periodic event |
| • make predictions from the derived sinusoidal regression equation | • sketch transformations of a given graph |
| • relate values of $a$, $b$, and $d$ of $y = a \sin(bx + c) + d$ to given window settings | • choose appropriate window settings for a graphing calculator, based on given values for amplitude, period, and vertical shift |
| • determine the values of $a$, $b$, and $d$ in the equation $y = a \sin(bx + c) + d$ or in a given graph | |
| • recognize that when $c \neq 0$ a horizontal transformation occurs | |
| • enter data into a list or scatter plot, apply a best-fit model (either linear, quadratic, or exponential), and use the regression equation to make predictions | |
| • distinguish between patterns that are possibly fractals and those that are clearly not fractals | |
| • generate the next two iterations of a fractal pattern, given the original shape and a written and visual description of the pattern | |
| • describe, in writing or by listing a sequence of numbers, the patterns related to length of sides and to number of sides or number of vertices of fractal shapes | |
| • describe the numerical pattern for the perimeter of each shape in a two-dimensional fractal pattern | |
| • determine an algebraic expression describing the perimeter of the shapes in a fractal pattern | • give a rule for the change in area for the first few iterations of a fractal pattern |
| • generalize a fractal pattern using an algebraic expression that represents the patterns in length of sides, number of sides, or number of vertices of fractal shapes | • find the volume of the shapes for the first few iterations of a simple fractal pattern and make conjectures |
| • participate in and contribute toward the problem-solving process for problems that require the analysis of patterns and fractals studied in Applied Mathematics 30 | • complete the solution to problems that require the analysis of patterns and fractals studied in Applied Mathematics 30 |

## *Examples*

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled SE. Parts labelled SE are appropriate examples for students who achieve the *standard of excellence.*

*Use the following information to answer the next question.*

The height, $h$, in metres, of a point on a ferris wheel at time, $t$, in seconds, can be represented by a sinusoidal function in the form $h = a \sin(bt + c) + d$, as shown below.




**1.** The function that **best** describes the height of the point on the ferris wheel is

**A.** $h = 8 \sin(0.39t + 1.57) + 10$

**B.** $h = 8 \sin(0.39t - 1.57) + 10$

**C.** $h = 8 \sin(t + 4.02) + 10$

**D.** $h = 8 \sin(t - 4.02) + 10$

**Solution:**

Enter values into the list function on a graphing calculator and perform a sinusoidal regression.

$h = 8.02\ldots\sin(0.39\ldots t + {}^{-}1.57\ldots) + 10.02\ldots$

*Use the following information to answer the next question.*

**A Set of Data**

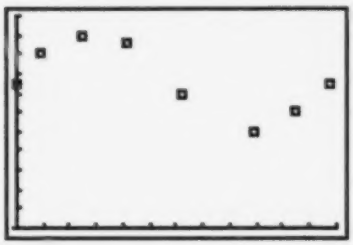
(0, 15)
(9, 18)
(25, 20)
(42, 19)
(62, 14)
(89, 10)
(104, 12)
(118, 15)

**2.** This data could most appropriately be modelled using a

**A.** linear regression
**B.** quadratic regression
**C.** sinusoidal regression
**D.** exponential regression

**Solution:**

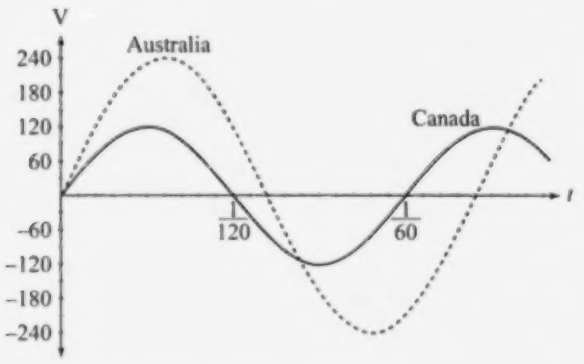
When plotted on a graphing calculator, the points form the following pattern:



This is clearly an example of data that models a sinusoidal function.

*Use the following information to answer the next question.*

The sinusoidal function that represents electrical current in Australia and the sinusoidal function that represents electrical current in Canada are graphed below.




**3.** The sinusoidal function representing electrical current in Australia differs from the sinusoidal function representing electrical current in Canada in

**A.** amplitude and period

**B.** period and horizontal shift

**C.** amplitude and vertical shift

**D.** horizontal shift and vertical shift

**Solution:**

The function representing electrical current in Australia takes longer to complete one cycle and has a higher maximum and lower minimum than the function representing electrical current in Canada. Therefore, the two graphs have different periods and amplitudes.

*Use the following information to answer the next three questions.*

The function $y = 17.14 \sin(0.48x - 1.75) + 7.11$, where $x$ represents the number of the month, models the average monthly temperature, in degrees Celsius, for Edmonton.

**4.** The amplitude of this function is

**A.** 0.48°C
**B.** 1.75°C
**C.** 7.11°C
**D.** 17.14°C

**Solution:**

The amplitude of a sinusoidal function in the form $y = a \cdot \sin(bx + c) + d$ is equal to the value of $a$ (in this case 17.14).

**5.** The maximum average temperature in Edmonton can be found by adding 17.14°C and

**A.** 7.11°C
**B.** 0.48°C
**C.** 1.75°C
**D.** –1.75°C

**Solution:**

The maximum value of a sinusoidal function can be found by adding 17.14 and 7.11.

**6.** Using her calculator set in radian mode, a tourist is able to determine that the average monthly temperature in Edmonton for month 8 (August) is

**A.** 1.8°C
**B.** 7.11°C
**C.** 22.0°C
**D.** 25.1°C

**Solution:**

For month 8, $x = 8$
$y = 17.14 \sin[0.48(8) - 1.75] + 7.11$
$y = 21.991\ldots$

7. The following table shows the average daily temperature by month in Winnipeg, Manitoba, over a period of one year.

| Average Daily Temperature in Winnipeg, Manitoba | |
|---|---|
| **Month** | **Average Temperature** |
| January | –18 |
| February | –18 |
| March | –11 |
| April | 0 |
| May | 11 |
| June | 18 |
| July | 20 |
| August | 19 |
| September | 14 |
| October | 7 |
| November | –8 |
| December | –17 |

**a.** Use a graphing calculator to plot the data.

**b.** Determine an approximate sinusoidal function for this data. Express the values to the nearest hundredth. Graph this function in the viewing window provided and state the window settings.

$x$: [ , , ]

$y$: [ , , ]

function: _____

**c.** Use the regression model to determine the average temperature on August 15.

**d.** What is the period of this function? How does it compare with your knowledge of annual temperature changes?

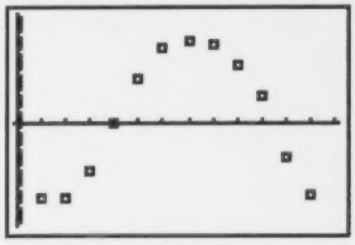
**e.** What do the parameters *a* and *d* in your regression equation represent in terms of the annual mean temperature and range of temperatures in Winnipeg, Manitoba?

**f.** The growing season is the part of the year in which the average temperature remains above 5°C. What is the growing season in Winnipeg? Justify your answer by using data from the graph or equation above.

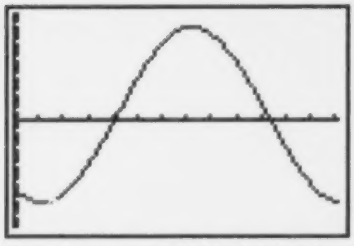
g. If the average daily temperature rose by 2°C as a result of global warming, what would be the effect on the sinusoidal function? What would be the new growing season? Explain.

**Solution:**

a. A possible solution



b.



$x: [\ 0,\ 13.1,\ 1\ ]$

$y: [\ -25,\ 25,\ 1\ ]$

function: $y = 20.61 \sin(0.52x - 2.16) + 1.21$

c. There are 31 days in August, so for August 15, $x = 8.48$ (approximately). When $x = 8.48$, $y = 17.5$. Therefore, the average temperature on August 15 is 17.5°C.

d. $\text{period} = \dfrac{2\pi}{b}$

$b = 0.51799...$

$\dfrac{2\pi}{0.51799...} = 12.1298...$

The period of this function is 12.1. This value seems reasonable since the temperatures should follow this pattern every year.

e. The value of $a$ is 20.60... and the value of $d$ is 1.20.... This indicates that the annual mean temperature in Winnipeg is about 1.21°C. The range of the temperature should be from as high as 21.82°C (1.21 + 20.61) to as low as –19.40°C (1.21 – 20.61).

f. Graph the line $y = 5$°C on the same axis as the regression equation. The sinusoidal curve lies above $y = 5$ between $x = 4.5$ and $x = 9.9$. These points translate to April 15 and September 27. Therefore, the growing season is from April 15 to September 27.

g. The value of $d$ in the sinusoidal function would increase by 2, so the new equation would be

$$y = 20.61 \sin(0.52x - 2.16) + \mathbf{3.21}$$

The points of intersection of this curve and $y = 5$ are $x = 4.3$ and $x = 10.1$. The new growing season would be from April 9 to October 3.

*Use the following information to answer the next question.*

A researcher discovered mould growing in a petri dish in her laboratory. When first observed, the mould covered only 12.5% of the dish's surface. After 24 hours, the surface area of the mould doubled in size, as shown in the table below.

| Time (h) | Area covered (%) |
|---|---|
| 0 | 12.5 |
| 24 | 25 |
| | |
| | |

**Numerical Response**

**8.** If the mould continues to grow at the same rate, the petri dish will be covered with mould after _____ hours.

(Record your answer in the numerical-response section on the answer sheet.)

**Solution:**

If the mould continues to double every 24 hours, the chart could be completed as follows:

| Time (h) | Area covered (%) |
|---|---|
| 0 | 12.5 |
| 24 | 25 |
| **48** | **50** |
| **72** | **100** |

The petri dish will be covered with mould after 72 hours.

*Use the following information to answer the next question.*

The median price for houses in Calgary for each two-year period from 1988 to 1998 is given below.

| Number of Years after 1988 | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| Price ($) | 127 526 | 127 621 | 132 868 | 134 643 | 157 353 | 176 316 |

These figures may be represented by an equation of the form $y = ax^2 + bx + c$.

**Numerical Response**

**9.** The value of $a$ in the equation above, rounded to the nearest whole number, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

**Solution:**

The median price for houses in Calgary over these 10 years can be expressed by a quadratic regression equation in the form

$$y = 733x^2 - 2545.41\ldots x + 128571.57\ldots$$

The value of $a$ in this equation is 733.

*Use the following information to answer the next question.*

The population in a particular high school was tracked over time.

| School Year | Population |
|---|---|
| 0 | 621 |
| 1 | 852 |
| 2 | |
| 3 | 1 250 |
| 4 | 1 324 |
| 5 | 1 348 |

A student used a quadratic regression equation to represent this data.

**10.** The student was able to estimate that the population of the school in year 2 was

**A.** 941
**B.** 1 051
**C.** 1 083
**D.** 1 085

**Solution:**

The data can be modelled by the quadratic regression equation
$y = -29.84\ldots x^2 + 298.25\ldots x + 607.96\ldots$.

Graph this equation and use the CALC function on the graphing calculator for $x = 2$. This function indicates that $y = 1085.09\ldots$. There were approximately 1 085 students in the school in year 2.

*Use the following information to answer the next question.*

A group of biologists studied the population of wolves in an area in northern Canada. The biologists found that the number of wolves was directly related to the number of caribou found in the study area. The wolf population for the period of the study is shown in the table below.

| Year | Wolf Population |
|---|---|
| Base year (0) | 500 |
| 1 | 548 |
| 2 | 800 |
| 3 | 1 168 |

**Written Response—10%**

**11.** **a.** State the exponential regression equation for this data in the form $y = ab^x$. Round the value of $a$ to the nearest whole number and the value of $b$ to the nearest hundredth.

**A POSSIBLE SOLUTION to part a**

$y = 392\ (1.43)^x$

**b.** The biologists found that a herd of approximately 17 800 caribou was needed to sustain 650 wolves. If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how many caribou would be required to sustain the wolf population in year 4?

**A POSSIBLE SOLUTION to part b**

**Using Rounded Values:**

$y = 392\ (1.43)^4$
$y = 1639.193476$

$$\frac{17\,800}{650} = \frac{\text{caribou}}{1639.193476}$$

Caribou = 44 888.68288

The caribou should number 44 889.

**Using Calculated Values:**

Graph the exponential regression and use the CALC function to determine the value of $y$ when $x = 4$.

$y = 1651.8014$

$$\frac{17\,800}{650} = \frac{\text{caribou}}{1651.8014}$$

Caribou = 45 233.94603

The caribou should number 45 234.

*Use the following additional information to answer the next part of the question.*

The area the biologists studied was approximately 14 500 km$^2$, and they found the maximum population density of wolves to be 18 wolves/100 km$^2$.

**c.** • What is the maximum number of wolves that this area can sustain?

**A POSSIBLE SOLUTION to part c, bullet 1**

$$\frac{14\,500}{100} \times 18 = 2\,610$$

The area can sustain 2 610 wolves.

• If the wolf population continued increasing at the same rate as it did in years 0 to 3 of the study, how much time will have elapsed when this maximum number of wolves is reached?

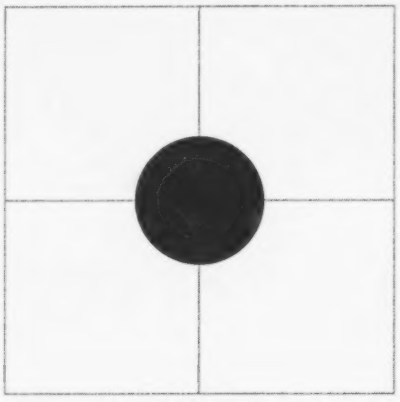
**A POSSIBLE SOLUTION to part c, bullet 2**

1. Graph $y = 392.44\ldots (1.43\ldots)^x$
2. Graph $y = 2610$
3. Use INTERSECT function on calculator when $y = 2\,610$, $x = 5.273\ldots$

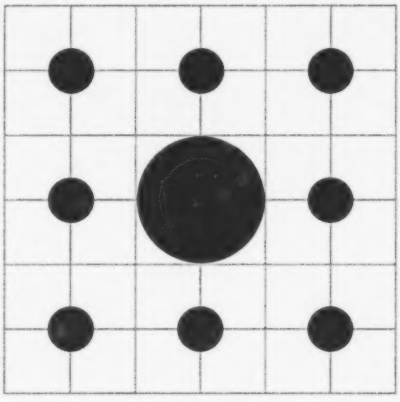
The wolf population will reach the maximum sustainable population during year 5.

*Use the following information to answer the next question.*

The diagrams below show the first two iterations of a fractal pattern. In each consecutive iteration, 8 new circles are drawn around the circles created in the previous iteration.



Iteration **1**



Iteration **2**

**12.** The total number of **new** circles created in iteration 3 will be

**A.** 17

**B.** 64

**C.** 72

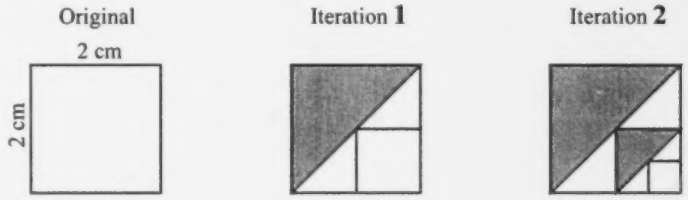
**D.** 81

**Solution:**

There were 8 new circles created in iteration 2. In iteration 3, each of these circles will have 8 new circles around it. This means that the total number of new circles created in iteration 3 will be 64.

*Use the following information to answer the next question.*

A student has a 2 cm by 2 cm square piece of paper. The student draws a diagonal of this square and shades the region of the square above this diagonal. In the unshaded region, the student then draws a second square by using the midpoint of the diagonal as one corner. This process produces iteration 1 shown below. The process is then continued with the smaller square, as shown in iteration 2.




**Numerical Response**

**13.** The total area of the shaded regions shown in the diagram for iteration 2, correct to the nearest tenth, is _____ $cm^2$.

(Record your answer in the numerical-response section on the answer sheet.)

**Solution:**

Area of larger shaded triangle: $A_1 = \frac{2 \times 2}{2} = 2\ cm^2$

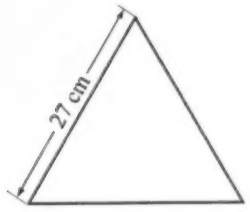
Area of smaller shaded triangle: $A_2 = \frac{1 \times 1}{2} = 0.5\ cm^2$

Total area: $2\ cm^2 + 0.5\ cm^2$

$= 2.5\ cm^2$

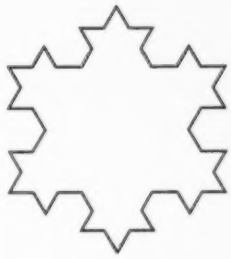
*Use the following information to answer the next two questions.*

The following diagrams show the original triangle and the first two iterations in the construction of a Koch snowflake. The original triangle is an equilateral triangle with 27 cm sides.




Iteration **0**
(Original)

Iteration **1**



Iteration **2**

The following chart shows the relationship between the added perimeter and the total perimeter for the Koch snowflake.

| Iteration | Added Perimeter (cm) | Total Perimeter (cm) |
|---|---|---|
| 0 | 0 | 81 |
| 1 | 27 | 108 |
| 2 | 36 | 144 |
| 3 | 48 | |

**Numerical Response**

**14.** The total perimeter of iteration 3, to the nearest centimetre, will be _____ cm.

(Record your answer in the numerical-response section on the answer sheet.)

**Solution:**

Total perimeter = 144 $cm^2$ + 48 $cm^2$
= 192 $cm^2$

*Use the following additional information to answer the next question.*

Exponential regression can be used to relate iteration number to **total** perimeter and to determine the perimeter of a particular iteration of this Koch snowflake.

**15.** Based on an exponential regression of the perimeters of iterations 0, 1, and 2, the perimeter, to the nearest centimetre, of iteration 10 will be

**A.** 1 918 cm

**B.** 1 081 cm

**C.** 1 438 cm

**D.** 123 cm

**Solution:**

Exponential regression equation: $y = 81(1.\overline{3})^x$

Iteration 10:[1] $y = 81(1.\overline{3})^{\,0}$

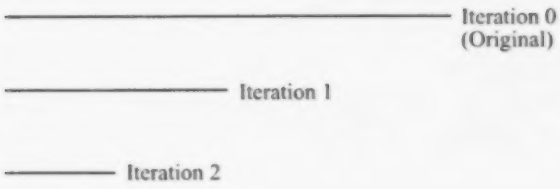
$y = 1438.37\ldots\text{cm}$

**16.** Determine which of the following patterns are examples of fractals. Explain your answer.

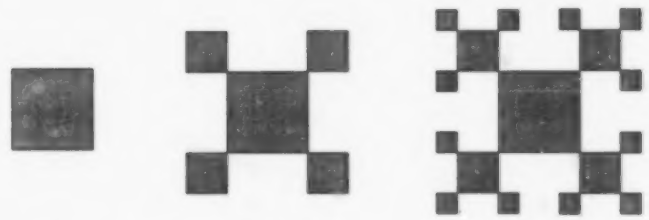
**a.**

Iteration 0 (Original) Iteration 1 Iteration 2

**b.**




**c.**



Iteration 0 (Original) Iteration 1 Iteration 2

**Solution**

**a.** This is an example of a fractal pattern. Each iteration is more complex and exhibits self-similarity.

**b.** This is not a fractal pattern. Each iteration is not more complex than the previous one.

**c.** This is not a fractal pattern. The iterations do not exhibit self-similarity.

## Vectors

### *General Outcome*

Solve problems involving polygons and vectors, including both 3-D and 2-D applications.

**General Notes:**

- Students should be reminded to ensure that their calculator is in degree mode before doing problems in this unit.
- Before starting the unit, it is strongly recommended that teachers review with their students Sine Law, Cosine Law, and right-angle trigonometry from Applied Mathematics 10 and bridge the trigonometry problems to appropriate vector terminology and diagrams.
- The concept of the alternate and corresponding angles related to transversal and parallel lines is crucial to succeed in this unit and will need to be reviewed.
- Students will need to understand the difference between scalar quantities, such as distance and speed, which identify only magnitude (e.g., 50 m, 100 km/h) and vector quantities, such as displacement and velocity, which identify direction as well as magnitude (e.g., 100 km/h in a direction 10° south of east).
- Teachers may want to make a connection with Physics 20 and Science 10 concepts. The mathematical process of visualization should be addressed when working with vectors. For example, when doing vector addition, students should draw the vectors. This can be done using technology, prebuilt vector programs, and/or student modelling of vectors of displacement, velocity, etc.
- *Geometer's Sketchpad*, *Calculator-Based Rangers*, isometric dot paper, and engineering paper may all be helpful tools in teaching this unit.
- Application problems will be limited to force, displacement, and velocity.
- The vector unit is based on applications of right-angle and oblique triangle trigonometry.
- Most problems are applications of the primary trigonometric ratios and the Cosine Law.
- In general, questions in which a diagram is provided will be deemed to be at the *acceptable standard*, and questions in which no diagram is given will be deemed to be at the *standard of excellence*.
- Angle measurements for direction of vectors will be dealt with in the context of the situation. Students should be prepared to work with both bearing notation (clockwise from north) and directional (heading) notation (W35° S means 35° south of west). We assume north to always be pointing to the top of the page. Since directional notation is not covered in the textbook, this area may need to be supplemented.
- Unless otherwise stated, angle measures should be stated to the nearest degree and resultant magnitudes to the nearest tenth.

## *Specific Outcomes*

**Specific Outcome 5.1**
Use appropriate terminology to describe:
- vectors quantities
- scalar quantities [C, CN]

**5.1 Note:**
- Use an arrow for direction and length of relative magnitude.

*Refer to examples 1 and 3.*

**Specific Outcome 5.2**
Assign meaning to the multiplication of a vector by a scalar. [CN]

**5.2 Note:**
- Students should be encouraged to use visualization and scale models when working with vectors.

*Refer to example 1.*

**Specific Outcome 5.3**
Determine the magnitude and direction of a resultant vector, using triangle or parallelogram methods. [CN, R, T, V]

**5.3 Notes:**
- Students should be comfortable using both parallelograms and triangles (head-to-tail) to solve problems involving vectors.
- When a force acts on a body, it should be represented by a vector whose initial point is the body and whose terminal point is in the direction that the force will move away from the body.
- Students will require a great deal of practice drawing appropriate vector diagrams and resultant vectors (both head-to-tail diagrams and parallelograms).
- The intent of the study of vectors in Applied Mathematics 30 is that students become aware that vectors represent real values, and therefore, resultant vectors should make sense physically.
- Students need to be made aware that the trigonometric solution they calculate for a problem may need to be **adapted** to the context of the vector problem.

*Refer to examples 2, 4, and 7.*

**Specific Outcome 5.4**

Model and solve problems in 2-D and simple 3-D, using vector diagrams and technology. [CN, PS, T, V]

**5.4 Notes:**

- Students at the *Acceptable Standard* should only be presented with illustrated 3-D problems that use multiple right triangles in two dimensions. Refer to example 9 for clarification.
- Teachers may wish to use rectangular prisms to help students model 3-D vectors.

*Refer to examples 5, 6, 8, 9, and 10.*

| *Acceptable Standard* | *Standard of Excellence* |
| --- | --- |
| The student can | The student can also |
| • identify properties of a parallelogram and use the properties to draw appropriate vector diagrams | |
| • construct an appropriate vector diagram from given information | |
| • describe vectors and scalar quantities by using appropriate terminology | |
| • distinguish between a scalar quantity and a vector quantity | |
| • determine relative magnitude of vectors by comparing lengths | |
| • recognize that multiplying a vector by a positive scalar changes magnitude only | |
| • recognize that multiplying a vector by a negative scalar reverses direction | |
| • solve problems involving scalar multiplication | |
| • calculate magnitude and direction of a resultant vector, given a diagram | • calculate magnitude and direction of a resultant vector without being given a diagram |
| • solve 2-D problems involving right triangles in which a diagram (but not a vector diagram) is given. | |
| • solve 2-D problems in which a vector diagram is given. | • solve 2-D problems without being given a parallelogram or head-to-tail diagram |
| • model 3-D problems | • model and solve 3-D problems<br>• solve 3-D problems that have been represented by two 2-D diagrams (one in a horizontal plane and one in a vertical plane) |
| • participate in and contribute toward the problem-solving process for problems that require the analysis of vectors studied in Applied Mathematics 30 | • complete the solution to problems that require the analysis of vectors studied in Applied Mathematics 30 |

## *Examples*

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled SE. Parts labelled SE are appropriate examples for students who achieve the *standard of excellence*.

**1.** The vector $\vec{a}$ represents a velocity of 40 km/h east. Make a scale drawing of each of the following vectors.

**a.** $3\vec{a}$

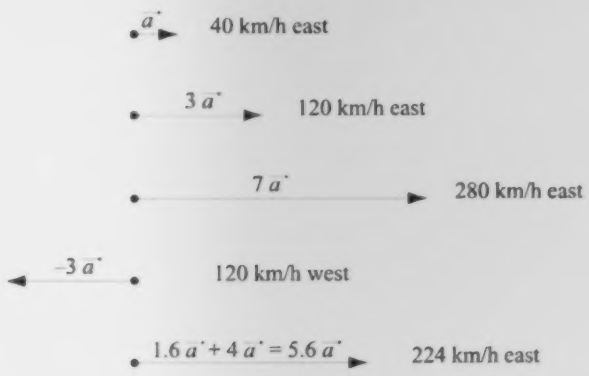
**b.** $7\vec{a}$

**c.** $-3\vec{a}$

**d.** $1.6\vec{a} + 4\vec{a}$

What velocities do each of these vectors represent?

**Solution:**



*Use the following information to answer the next question.*

The path of a particular vehicle is shown below.

2. The direction in which the vehicle is travelling is

   **A.** N30°W
   **B.** N60°W
   **C.** on a bearing of 60°
   **D.** on a bearing of 300°

**Solution:**

The vehicle is travelling in a direction of 60° north of west (or 30° west of north). This can be expressed as N30°W.

3. Which of the following statements describes a vector quantity?

   **A.** A car travelled north.

   **B.** A car travelled at 100 km/h.

   **C.** A car travelled north at 100 km/h.

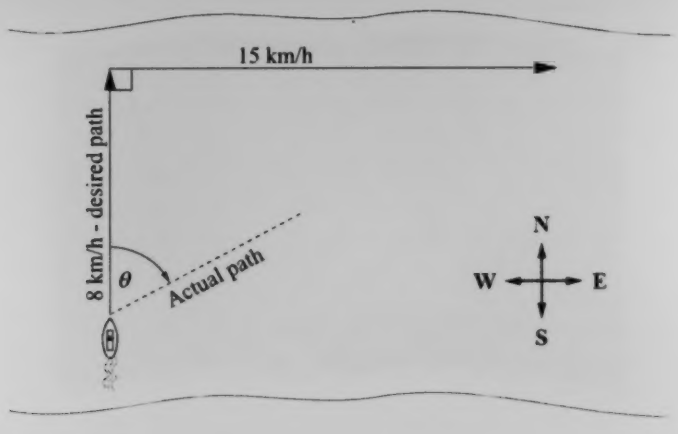
   **D.** A car travelled 200 km at 100 km/h.

**Solution:**

Vectors must have both magnitude and direction. The only response that meets these criteria is C.

*Use the following information to answer the next question.*

A person is trying to sail a boat north across a river, but the boat is being pushed off-course by a 15 km/h current flowing toward the east, as shown below.




4. *Relative to the shore, the actual speed of the boat is* ___*i*___ *km/h, and relative to the desired path, the boat is travelling off-course by an angle,* $\theta$, *of* ___*ii*___.

The statement above is completed by the information in row

| Row | *i* | *ii* |
|---|---|---|
| **A.** | 17 | 62° |
| **B.** | 17 | 28° |
| **C.** | 13 | 62° |
| **D.** | 13 | 28° |

**Solution:**

$\sqrt{8^2+15^2}$ = actual path ($i$)

$17 = i$

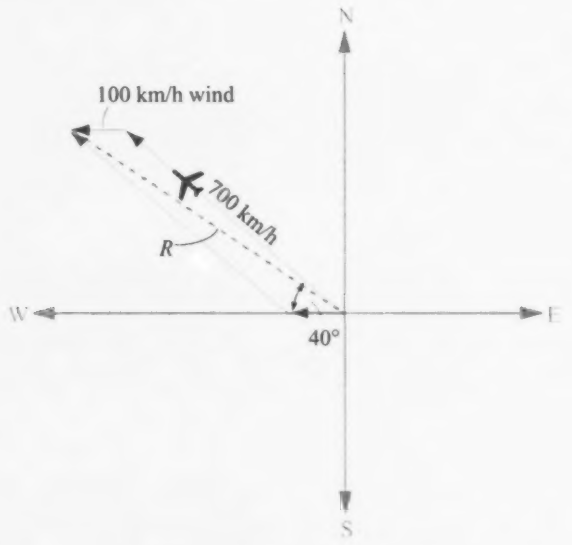
$\tan\theta = \frac{15}{8}$

$\theta \doteq 62^\circ$

Relative to the shore, the actual speed of the boat is ___17___ km/h, and relative to the desired path, the boat is travelling off-course by an angle, $\theta$, of ___62°___.

*Use the following information to answer the next question.*

A 737 aircraft is flying on a bearing of 310° with an air speed of 700 km/h. It is being affected by a 100 km/h wind blowing from the east, as shown in the diagram below.




**5.** The distance, $R$, relative to the ground, that the plane has travelled after 1 hour is, to the nearest kilometre,

**A.** 627 km

**B.** 640 km

**C.** 707 km

**D.** 779 km

**Solution:**

The angle between the two labelled vectors is 140°.

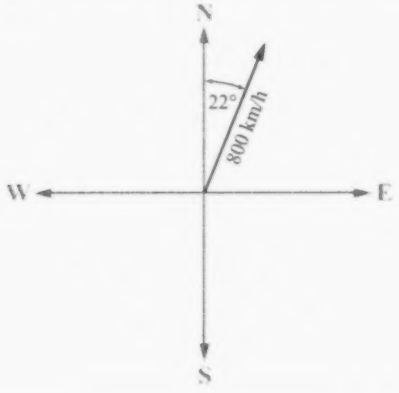
$R^2 = 700^2 + 100^2 - 2(700)(100)(\cos 140°)$

$R = 779.2600\ldots$

After 1 hour, the plane has travelled 779 km.

*Use the following information to answer the next question.*

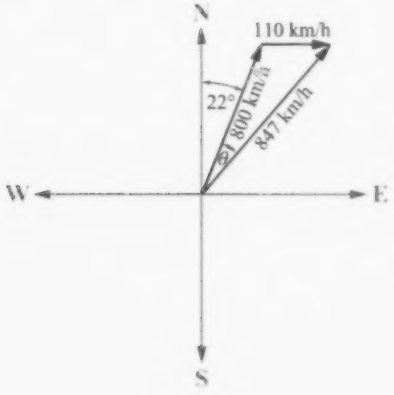
An aircraft sets a course with a bearing of 22° and a speed of 800 km/h, as shown in the diagram below. A horizontal wind blowing from the west at a speed of 110 km/h causes the aircraft to fly off-course with a speed of 847 km/h by an angle $\theta$.




SE **6.** The angle, $\theta$, by which the aircraft is flying off-course is, to the nearest degree,

**A.** 7°
**B.** 22°
**C.** 61°
**D.** 68°

**Solution:**

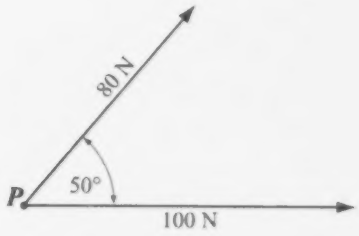



$$\cos\theta = \frac{847^2 + 800^2 - 110^2}{2(847)(800)}$$
$$\cos\theta = 0.9927...$$
$$\theta = 6.9266...$$

The aircraft is flying off-course at an angle of 7°

*Use the following information to answer the next question.*

Two forces act on a point $P$. The first force has a magnitude of 100 N. The second force has a magnitude of 80 N and is at an angle of 50° to the first force, as shown below.

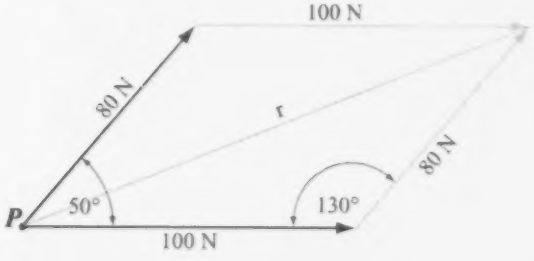



SE **7.** The magnitude of the resultant force, to the nearest newton, is

**A.** 31 N
**B.** 128 N
**C.** 163 N
**D.** 180 N

**Solution:**

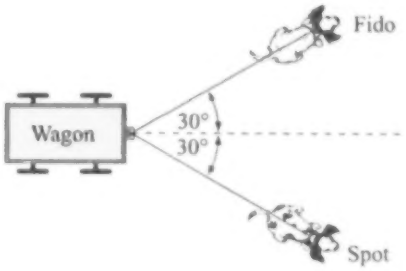
Sketch an appropriate parallelogram diagram and solve for $r$.




$r^2 = 100^2 + 80^2 - 2(100)(80)\cos 130°$
$r^2 = 26\,684.60175$
$r = 163.33542...$
The magnitude of the resultant force is 163N.

*Use the following information to answer the next question.*

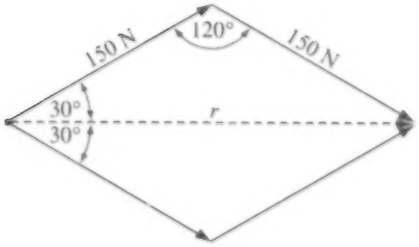
Two dogs are pulling a wagon. Each dog exerts a force of 150 N. Each dog is pulling in a direction that is 30° away from the direction of travel, as shown below.




SE **8.** The resultant force in the direction of travel, to the nearest newton, is

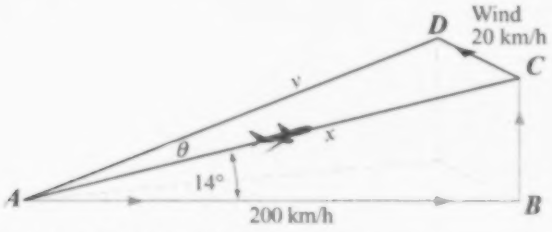
**A.** 75 N
**B.** 150 N
**C.** 260 N
**D.** 290 N

**Solution:**




$r = \sqrt{150^2 + 150^2 - 2(150)(150)\cos 120^\circ}$
$r = 259.8076...$
The resultant force is 260N.

SE **9.** A plane sets a course to fly with a ground speed of 200 km/h due east while climbing at an angle of 14°. Upon takeoff, it is affected by a horizontal 20 km/h wind blowing directly north, as shown in the diagram below.



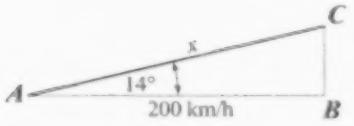

**a.** Model this problem using two 2-D diagrams.

**b.** Calculate the relative velocity ($v$) of the plane and calculate the direction ($\theta$) that the plane is flying off its course.

**c.** What course correction should the pilot set to remain on track?

**Solution**

**a.**



**b.**

$$\cos 14° = \frac{200}{x}$$

$$x = \frac{200}{\cos 14°}$$

$$x = 206.1227\ldots \text{ km/h}$$

$$\tan\theta = \frac{20}{206.1227\ldots}$$

$$\theta = 5.5°$$

$$v = \sqrt{x^2 + (20)^2}$$

$$v = \sqrt{(206.1227\ldots)^2 + (20)^2}$$

$$v = 207.09\ldots$$

The plane is flying at a relative velocity of 207.1 km/h and is off course by a bearing of 84.5°.

**c.** To correct the path, the pilot should set the course correction on a bearing of 95.5°.

*Use the following information to answer the next question.*

As Norma walks into her office tower, she is facing north. She walks due east from the front door to the elevator. She takes the elevator up 10 floors, then walks due north to her office.

**10.** Which of the following 3-D diagrams could be used to model Norma's path?

**A.**

**B.**

**C.**

**D.**

**Soultion:**

The 3-D diagram that correctly illustrates Norma's path is C.

## Statistics and Probability

### *General Outcomes*

Use normal and binomial probability distributions to solve problems involving uncertainty.

Model the probability of a compound event, and solve problems based on the combining of simpler probabilities.

**General Notes:**
- Students may need to review basic concepts and terms of probability.
- Teachers may want to do the probability section of this unit first.
- Probability should be expressed as a decimal or fractional value between 0 and 1.

### *Specific Outcomes*

**Specific Outcome 2.1**
Find the population standard deviation of a data set, using technology. [CN, E, T, V]

**2.1 Notes:**
- Students are also expected to have an understanding of what standard deviation is.
- Review measures of central tendency and histograms. Teachers may also need to review the difference between a sample and a population.
- Sample standard deviation will not be used. Population standard deviation is represented by $\sigma$.
- Students must be familiar with the procedures required to input values into the list function on their calculator.

*Refer to examples 1 and 4.*

**Specific Outcome 2.2**
Use $z$-scores to solve problems related to the normal distribution. [PS, R, T, V]

**2.2 Notes:**
- Some graphing calculators on the approved list do not have the capability to graph $z$-score probability curves; with these models of calculators, tables must be used.
- Students should have practice with paper and pencil calculations and sketches before being introduced to technology.
- Teachers and students need to be aware that the $z$-scores and areas under the curve from the textbook and data table may vary slightly from those values given by the calculator. When writing diploma examinations, students may use either technology **or** $z$-score tables.

*Refer to examples 3, 5, and 9.*

**Specific Outcome 2.3**
Use the normal approximation to the binomial distribution to solve problems involving confidence intervals for large-sample binomial experiments. [CN, E, PS, T]

**2.3 Notes:**
- A sample is considered to be large if $np > 5$ and if $n(1-p) > 5$. Continuity correction will **not** be assessed on the diploma examination.
- This outcome addresses only **discrete** data and is not intended for continuous variables.
- Students are expected to calculate 95% confidence intervals only. However, teachers should discuss with students the differences between 90%, 95%, and 99% confidence intervals.
- **Calculation** of the margin of error is **not** included in this outcome. However, an understanding of how margin of error relates to the size of the confidence interval may be beneficial.

*Refer of examples 2, 6, 7, 8c, and 8d.*

**Specific Outcome 2.4**
Construct a sample space for two or three events. [PS, R, V]

**2.4 Notes:**
- Some discrete sample spaces can also be displayed as the intersection points on a two-dimensional grid.
- Once a sample space has been constructed, students should be able to answer questions based on it by using independent probability calculations or through investigation.

*Refer to examples 8a and 11.*

**Specific Outcome 2.5**
Classify events as independent or dependent. [C]

**2.5 Note:**
- The distinction between independent and dependent events should be approached through visualization of experimental design rather than through calculations.

*Refer to example 10.*

**Specific Outcome 2.6**
Use expressions for $P(A \text{ and } B)$ to solve problems involving independent and dependent events. [CN, PS, R]

*Refer to example 13.*

**Specific Outcome 2.7**
Solve problems using the probabilities of mutually exclusive and complementary events. [CN, PS, R]

*Refer to examples 12 and 14.*

**2.7 Notes:**
- The formulas for $P(A \text{ or } B)$ should be restricted to the mutually exclusive case only.
- Non-mutually exclusive events are beyond the scope of Applied Mathematics 30.

*Refer to example 8b.*

| *Acceptable Standard* | *Standard of Excellence* |
|---|---|
| The student can | The student can also |
| • determine population standard deviation and mean using hand-held technology | |
| • use technology to create a histogram, and compare it with a normal distribution | |
| • given various histograms, rank the standard deviation without doing any calculation | • interpret how changing the data can affect the standard deviation and/or mean |
| • calculate a $z$-score, using the formula | |
| • calculate the missing value when given the value of 3 of the 4 variables in the $z$-score formula | |
| • sketch a diagram of a normal curve and indicate appropriate shading for a given problem | |
| • given the values of $n$ and $p$, calculate a $z$-score and apply it to the normal curve | • use $z$-scores to compare two sets of data and draw conclusions |
| • calculate the area under the standard normal curve to the left of a $z$-score | • calculate the area to the right of a $z$-score or between two $z$-scores |
| • describe the properties of a normal distribution | • make inferences given an area under the standard normal curve |
| • given a binomial distribution and using appropriate formulas, calculate mean and standard deviation | |
| • given binomial distribution data and the values for $\mu$ and $\sigma$, calculate the $z$-score or symmetric 95% confidence interval | • given binomial distribution data, extract the values of $n$ and $p$ in a given context to calculate a $z$-score or symmetric 95% confidence interval, and apply it to the normal curve |
| • given the values of $n$ and $p$, calculate a $z$-score and apply it to the normal curve | |
| • determine sample spaces for problems involving two or three components | • generate a sample space and use it to draw conclusions about the outcomes of non-routine problems |
| • given a sample space, draw conclusions about the outcomes of routine problems | |
| • distinguish between independent and dependent events | • given the probability of one event and the probability of combined events, determine the probability of the other event |
| • determine $P(A$ and $B)$ for independent events | |
| • determine $P(A$ and $B)$ for dependent events where order is specified | |
| • determine $P(A$ or $B)$ for single events that are mutually exclusive | |

| | |
|---|---|
| • identify complementary events<br>• determine the complement to a particular event<br>• given the probability of an event, determine the probability of the complement<br>• participate in and contribute toward the problem-solving process for problems that require the analysis of statistics and probability studied in Applied Mathematics 30 | • complete the solution to problems that require the analysis of statistics and probability studied in Applied Mathematics 30 |

## *Examples*

*Use the following information to answer the next question.*

The average individual score per round for each of several golfers on the 2000 PGA tour is recorded in the table below.

| Name | Average | Name | Average |
|---|---|---|---|
| Mike Weir | 70.4 | Joe Durrant | 70.9 |
| Greg Chalmers | 70.5 | Tiger Woods | 67.8 |
| David Duval | 69.4 | Scott Dunlop | 70.4 |

**Numerical Response**

**1.** The standard deviation, $\sigma$, of the golfers' average scores, to the nearest hundredth, is _____.

**Solution:**

Enter the values into the list function on a graphing calculator and use the single variable statistics function to calculate the population standard deviation $\sigma = 1.04$

*Use the following information to answer the next question.*

A potato chip manufacturing company has found that the mean mass of potato chips in its bags is 65 g and the standard deviation is 7 g.

**Numerical Response**

**2.** The symmetric 95% confidence interval for the mass of potato chips in a bag, to the nearest whole number, is between 51 g and _____ g.

**Solution:**

| **Method 1** | | **Method 2** |
|---|---|---|
| Upper bound | $= 65 + 1.96(7)$ | $65 - 51 = 14$ |
| | $= 78.72$ | $65 + 14 = 79$ |

The symmetric 95% confidence interval for the mass of potato chips in a bag, to the nearest whole number, is between 51 g and 79 g.

*Use the following information to answer the next question.*

A medical researcher measured the body temperature of 700 people and found that the temperatures were normally distributed with a mean of 36.8° C and a standard deviation of 0.35° C.

**3.** The number of people expected to have a body temperature of 37.5° C or lower is

**A.** 16

**B.** 68

**C.** 490

**D.** 684

**Solution:**

**Method 1**

Normalcdf (0, 37.5, 36.8, 0.35)
$= 0.97724\ldots$
$700 \times 0.97724\ldots = 684.07\ldots$

**Method 2**

$$z = \frac{37.5 - 36.8}{0.35}$$
$z = 2$
$P(z \leq 2) = 0.9772$
$0.9772 \times 700 = 684.04$

The number of people expected to have a body temperature of 37.5°C or lower is 684.

*Use the following information to answer the next question.*

Henry played 24 golf games on the same course during each of two seasons. In the first season, his mean score was 78 with a standard deviation of 2.1. In the second season, his mean score was 74 with a standard deviation of 3.8.

**4.** The **standard deviation** of Henry's scores for the two seasons indicates that his

**A.** scores were more consistent in the first season

**B.** scores were more consistent in the second season

**C.** average score was better in the first season

**D.** average score was better in the second season

**Solution:**

Standard deviation is a measure of the extent to which the data are spread from the mean. Since Henry's second season scores have a larger standard deviation, his scores in the first season were more consistent.

SE **5.** In a particular town, 70% of the students are bused to school. In a random sample of 1 000 students, the mean of the number of students bused to school is expected to be 700, with a standard deviation of 14.49. The probability that in any given sample of 1 000 students, 720 or more students are bused to school is

**A.** 0.08
**B.** 0.38
**C.** 0.62
**D.** 0.92

**Solution:**

**Method 1**

$x = 720$

$\mu = 700$

$\sigma = 14.49$

$$z = \frac{720 - 700}{14.49}$$

$z = 1.132$

$P(z \geq 1.32) = 1 - 0.9162$

$= 0.0838$

**Method 2**

Normalcdf (720, 1 000, 700, 14.49)

$= 0.0837....$

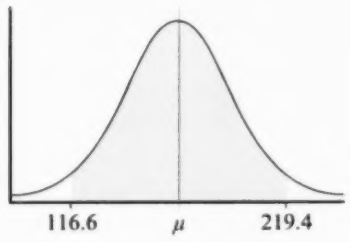
The probability that 720 or more students are bused to school is 0.08.

*Use the following information to answer the next question.*

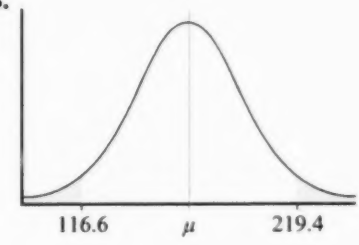
During a quality-control test, 20 batteries are picked at random from an assembly line. The symmetric 95% confidence interval for the average lifespan of each battery is 116.6 h to 219.4 h.

**6.** Which of the following graphs has a shaded region indicating this symmetric 95% confidence interval?

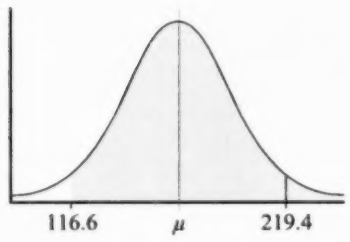
**A.**



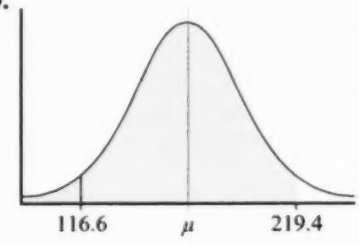

**B.**




**C.**




**D.**




**Solution:**

The symmetric 95% confidence interval includes all of the data that have a $z$-score between $-1.96$ and 1.96. Therefore the correct response is A.

*Use the following information to answer the next question.*

In a particular city with 50 000 high school students, 21 500 were found to be in favour of starting school earlier in the day in order to finish earlier. Josh surveys 100 randomly selected students from this population to determine if they are in favour of this proposal.

**SE** **7.** The symmetric 95% confidence interval for the number of students that Josh finds to be in favour of the proposal is

**A.** 3 to 83
**B.** 33 to 53
**C.** 36 to 50
**D.** 41 to 45

**Solution:**

95% confidence interval $= \mu \pm 1.96\sigma$

$\mu = n \cdot p$ $\qquad$ $\sigma = \sqrt{n \cdot p\,(1-p)}$

$\mu = 100\left(\frac{21\,500}{50\,000}\right)$ $\qquad$ $\sigma = \sqrt{100\,(0.43)(0.57)}$

$\mu = 43$ $\qquad$ $\sigma = 4.9507...$

Lower Bound $= \mu - 1.96\sigma$
$= 43 - 1.96\,(4.9507...)$
$= 33.296...$

Upper Bound $= \mu + 1.96\sigma$
$= 43 + 1.96\,(4.9507...)$
$= 52.703...$

The symmetric 95% confidence interval for the number of students that Josh finds to be in favour of the proposal is 33 to 53.

*Use the following information to answer the next question.*

In humans, some characteristics are genetically determined by genes, which occur in pairs. Each pair consists of any combination of dominant ($E$) and recessive ($e$) genes. For example, a person may have two dominant genes ($EE$), two recessive genes ($ee$), or a dominant and a recessive gene ($Ee$).

If a person has one or two dominant genes, then the detached earlobe characteristic is displayed.

Attached ear-lobe

Detached ear-lobe

**Written Response—10%**

**8.** **a.** Complete the Punnet square below to show the sample space for the offspring of two parents who both carry one dominant ($E$) gene for the detached earlobe.

**A POSSIBLE SOLUTION to part a**

| | | Mother | |
|---|---|---|---|
| | | ***E*** | ***e*** |
| Father | ***E*** | ***EE*** | ***Ee*** |
| | ***e*** | ***Ee*** | ***ee*** |

**b.** What is the probability that one offspring from these parents will have detached earlobes?

**A POSSIBLE SOLUTION to part b**

The probability that one offspring from these parents has detached earlobes is

$\frac{3}{4}$ or 0.75

*Use the following information to answer the next part of the question.*

Approximately 39% of the population exhibits the attached earlobe characteristic.

**c.** Use the binomial approximation to calculate the mean and standard deviation for the number of people in a sample of 8 748 that will have attached earlobes. Round your answers to the nearest hundredth.

**A POSSIBLE SOLUTION to part c**

$n = 8\,748$ $\quad \mu = np$ $\quad \sigma = \sqrt{np(1-p)}$

$p = 0.39$ $\quad \mu = 8748(.39)$ $\quad \sigma = \sqrt{8748(.39)(.61)}$

$q = 0.61$ $\quad \mu = 3411.72$ $\quad \sigma = 45.62$

The mean is 3 411.72 and the standard deviation is 45.62.

**d.** Calculate the 95% confidence interval for the number of people in this sample that will have attached earlobes.

**A POSSIBLE SOLUTION to part d**

95% confidence int. $= \mu \pm 1.96\sigma$

$= 3411.72 \pm 1.96(45.62)$

$= 3411.72 \pm 89.4152$

$= 3\,322.3048, 3\,501.1352$

It can be predicted, with 95% confidence, that between 3 322 and 3 502 people in this sample will have attached earlobes.

SE **9.** In the general population, the IQ scores of individuals are normally distributed with a mean of 110 and a standard deviation of 10. If a large group of people is tested,

**a.** what proportion will be expected to have IQs between 100 and 120?

**b.** what is the probability that an individual in the group has an IQ greater than 120?

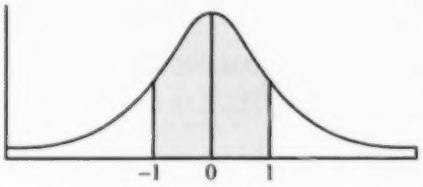
**c.** what IQ is necessary to be within the top 5% of the sample group?

**Solution:**

**a.**

$$z = \frac{120 - 110}{10} = 1$$

$$P(s < 120) = 0.8413$$




$$z = \frac{100 - 110}{10} = -1$$

$$P(s < 100) = 0.1587$$

$$\therefore P(100 < s < 120) = 0.8413 - 0.1587$$

$$P(100 < s < 120) = 0.6826$$

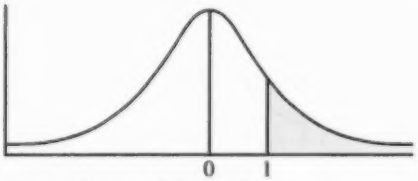
The proportion of this group with an IQ between 100 and 120 is 0.6826.

**b.**

$$P(s < 120) = 0.8413$$

$$P(s > 120) = 1 - 0.8413$$

$$\therefore = 0.1587$$




The probability of a group member having an IQ greater than 120 is 0.1587.

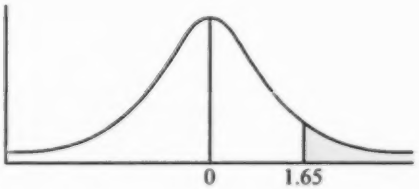
**c.**

$$P(s < z) = 0.95$$

$$z = 1.65$$

$$1.65 = \frac{x - 110}{10}$$

$$x = 126.5$$




The IQ necessary to be within the top 5% is 126.5.

*Use the following information to answer the next question.*

A box contains 6 blue balls and 4 red balls. Two balls are drawn from the box, one after the other, without replacement.

**10.** The actions described above will result in events that are

**A.** dependent

**B.** independent

**C.** complementary

**D.** mutually exclusive

**Solution:**

Since the first ball is not replaced before the second one is drawn, the probability of the second event depends on the results of the first event. The actions result in events that are dependent.

*Use the following information to answer the next question.*

The numbers 1 through 5 are each written on a separate slip of paper, and the papers are placed in a box. The letters A, B, C, and D are each written on a separate slip of paper, and the papers are placed into a **different** box. Jodi draws one slip of paper from each box.

**11.** The number of elements in the sample space for this trial is

**A.** 51

**B.** 20

**C.** 9

**D.** 2

**Solution:**

There are 5 numbers and 4 letters. Therefore, there are 20 elements in the sample space.

1A 2A 3A 4A 5A
1B 2B 3B 4B 5B
1C 2C 3C 4C 5C
1D 2D 3D 4D 5D

*Use the following information to answer the next question.*

A particular traffic light at the outskirts of a town is red for 30 s, green for 25 s, and yellow for 5 s in every minute.

**12.** The probability that the traffic light will **not** be green when a motorist first sees it is

**A.** $\frac{1}{2}$

**B.** $\frac{1}{12}$

**C.** $\frac{5}{12}$

**D.** $\frac{7}{12}$

**Solution:**

$$\text{P(not green)} = 1 - \text{P(green)}$$

$$\text{P(Green)} = \frac{25}{60}$$

$$= \frac{5}{12}$$

$$1 - \text{P(Green)} = 1 - \frac{5}{12}$$

$$= \frac{7}{12}$$

*Use the following information to answer the next question.*

Given their previous performance, the probability of a particular baseball team winning any given game is $\frac{4}{5}$.

**13.** The probability that the team will win their next 2 games is

**A.** $\frac{8}{5}$

**B.** $\frac{16}{25}$

**C.** $\frac{2}{5}$

**D.** $\frac{1}{25}$

**Solution:**

$$\begin{aligned} P(\text{winning next 2 games}) &= P(\text{winning next game}) \times P(\text{winning second game}) \\ &= \frac{4}{5} \bullet \frac{4}{5} \\ &= \frac{16}{25} \end{aligned}$$

*Use the following information to answer the next question.*

Malaga, Spain, lies in a region of Europe known as the Costa Del Sol (Coast of the Sun). The probability of sunshine on any given day in this region is approximately 0.89.

**Numerical Response**

**14.** In a non-leap year of 365 days, the average number of days of the year that a tourist could expect to experience weather **other** than sunshine, to the nearest whole number, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

**Solution:**

Probability of weather other than sunshine = 1 – 0.89
= 0.11

$\mu = np$
$\mu = 365(0.11)$
$\mu = 40$

A tourist should expect weather other than sunshine on 40 days a year.

## Design

### *General Outcome*

Analyze objects, shapes and processes to solve cost and design problems.

**General Notes:**

- The objectives for this unit are covered in the project book, **not** the textbook.
- Students have done considerable work with perimeter, area, scaling, and volume in Applied Mathematics 10 and 20.
- This unit should concentrate on the use of dimensions, unit prices, and estimation to cost projects.
- This unit should include projects wherein a design is given, projects wherein a design is not given, projects wherein a budget is specified, and projects wherein a budget is not specified.
- Generally, the difference between the student at the *acceptable standard* and the student at the *standard of excellence* is the quality of the solution and the extent to which mathematical processes are demonstrated.
- Outcomes 6.1 and 6.2 are primarily review and are not intended to be the main focus of this unit. They are more of a preparation for outcomes 6.3 and 6.4.
- Be aware of the amount of time spent on this unit—the general outcome is large because of its exploratory nature.

## *Specific Outcomes*

**Specific Outcome 6.1**
Use dimensions and unit prices to solve problems involving perimeter, area and volume. [E, PS, V]

*Refer to examples 3 and 4.*

**Specific Outcome 6.2**
Solve problems involving estimation and cost for objects, shapes or processes when a design is given. [C, E, PS]

*Refer to examples 1 and 5.*

**Specific Outcome 6.3**
Use appropriate variables to design an object, shape, layout or process within a specified budget. [C, PS, R, V]

**6.3 Note:**
- In this outcome, skills from many other areas may be required. Some problems may require various function types on the regression menu of the graphing calculator. It is better to do a few large design projects rather than many small questions.

*Refer to examples 2 and 6.*

**Specific Outcome 6.4**
Use mathematical models to estimate the solutions to complex measurement problems. [E, V]

*Refer to example 7.*

| *Acceptable Standard* | *Standard of Excellence* |
|---|---|
| The student can | The student can also |
| • calculate perimeter, surface area, and volume, and can determine costs of composite designs when calculations (i.e., Pythagorean Theorem, trigonometry, solving for any variable in familiar formulas, etc.) are required to find values of necessary dimensions | |
| • draw sketches and scale diagrams in 2-D from various views when a written description of a design is given | • draw sketches and scale diagrams in 3-D from various views when a written description of a design is given |
| • solve cost and estimation problems related to perimeter, surface area, and volume consistent with a diagram he or she has drawn | • communicate effectively the meaning from his or her solutions and justify the procedures used |
| • design objects based on previously learned mathematics by using given information | |
| • make allowances for changes in given problems | |
| • describe an estimation process that yields a reasonable result | • design and communicate the most efficient process to yield a desired result |
| • participate in and contribute toward the problem-solving process for problems that require the analysis of design studied in Applied Mathematics 30 | • complete the solution to problems that require the analysis of design studied in Applied Mathematics 30 |

## *Examples*

Students who achieve the *acceptable standard* should be able to answer all the following questions, except for any part of a question labelled [SE]. Parts labelled [SE] are appropriate examples for students who achieve the *standard of excellence*.

*The following example could be used as a project.*

**1.** A manufacturer of cylindrical cans uses tin plate that comes in sheets that are 240 cm by 160 cm and cost \$3.20 per sheet. Cans are 6 cm in diameter and 11 cm high, and they have 3 seals each. Seals cost 0.8¢ each to make.

Currently, the manufacturer uses two sheets to make sides and one sheet to make the end pieces.

**a.** How many ends can be made from one sheet?

**b.** How many sides can be made from two sheets?

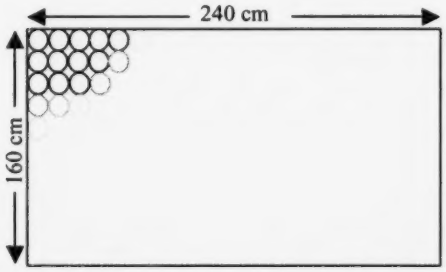
**c.** Use your answers from part a and part b to determine how many cans can be made from three sheets.

**d.** How much does it cost to make the cans using this method?

[SE] **e.** Design another method for using the three sheets to produce these cans. Is it more or less efficient than the current one? Explain.

**Solution:**

**a.**



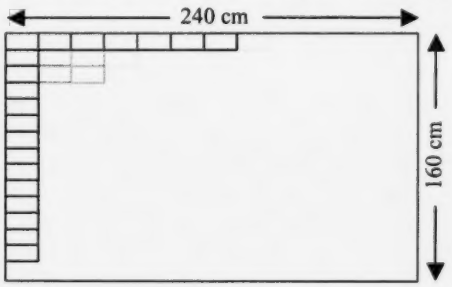

$\frac{240}{6} = 40$

$\frac{160}{6} = 26.\overline{6}$

$26 \times 40 = 1\,040$ ends

This is enough for 520 cans.

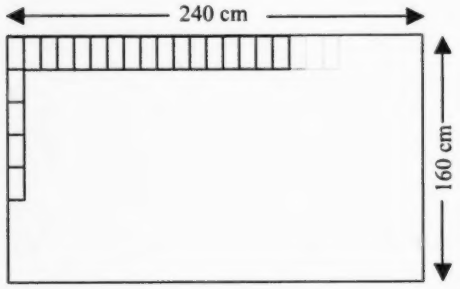
**b.** Since the cylindrical can has a circumference of $6\pi$ and a height of 11 cm, each side can be made using rectangles that are $6\pi$ cm $\times$ 11 cm. There are two ways that these rectangles can be placed on the sheet of tin. The first way is with the longer sides of the rectangles along the longer side of the sheet, as shown below.



With this method, the following number of sides can be made.

$$\left.\begin{aligned}\frac{160}{11} &= 14.55\\ \frac{240}{6\pi} &= 12.7\end{aligned}\right\} \quad \begin{aligned}14 \times 12 &= 168 \text{ sides/sheet}\\ \times 2 &\text{ sheets}\\ &= 336 \text{ sides can be made}\end{aligned}$$

The other way is to put the longer sides of the rectangles along the shorter side of the sheet, as shown below.



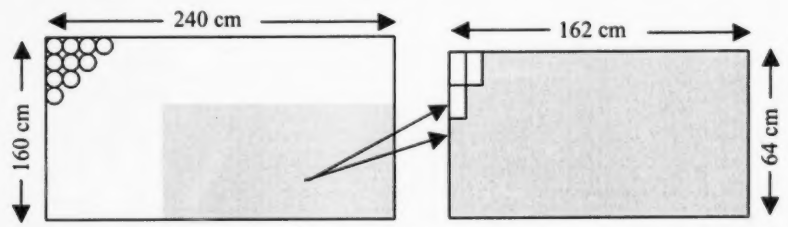

With this method, 336 cans can also be made.

$$\left.\begin{aligned}\frac{160}{6\pi} &= 8.49\\ \frac{240}{11} &= 21.8\end{aligned}\right\} 8 \times 21 \times 2 = 336$$

**c.** There are enough ends to make 520 cans but only enough sides to make 336; therefore, 336 cans can be made using this method.

**d.** $C = \$3.20 \times 3 + 3 \times 336 \times 0.008$
$= \$9.60 + \$8.06$
$= \$17.66$ to make 336 cans
The cost per can is \$0.052.

SE **e.** Since a significant amount of the sheet used for the ends is not used in the above method, part of that sheet could be used to make more sides.



From the unused part of the sheet, you can make an additional 42 sides and an additional 98 ends. This will make a total of 378 sides and a total of 770 ends.

This means you can make a total of 378 cans if you use this method.

The cost for this method is as follows:

| | |
|---|---|
| Cost for sheets | $\$3.20 \times 3 = \$9.60$ |
| Cost for seals | $\$0.008 \times 3 \times 378 = \$9.07$ |
| Total cost | \$18.67 |
| Cost per can | $\frac{\$18.67}{378} = \$0.049$ |

The cost is about the same for this method. However, since this method costs slightly less, over time, the difference in cost may prove significant.

*The following example could be used as a project.*

**2.** A window cleaner has been asked by the management of a large office building to submit a quotation for cleaning the building's windows. The office building has

- 24 floors
- 14 windows per floor, on each side of the building
- 4 sides

To clean the windows, the cleaner starts at the top of one column of windows and works his way to the bottom of that column, cleaning each window on the way down. He then goes back to the top, moves to the second column, and repeats the process. He does this for each side of the building.

From experience, the cleaner knows that it takes

- 120 seconds to clean one window
- 30 seconds to transfer between floors on the same column
- 120 seconds to go back to the top of the building once he has reached the bottom
- 60 seconds to transfer from one column to the next on the same side of the building
- 120 seconds to transfer from one side of the building to the next

**a.** How many hours will it take to wash all the windows in the building?

**b.** For every three hours that the window cleaner works, he takes a half-hour break. How many hours will it be before he finishes the entire building?

**c.** The window washer charges \$25 per hour (including the half-hour breaks). He has a base charge of \$120, and he wishes to add an additional 10% to the cost of this project so that he can reinvest in his business. Determine the quotation he should give the management of this building.

**Solution:**

**a.** Determine the time required to complete one side of the building.

First, determine the time required to complete one column of windows.

| | |
|---|---|
| 24 windows @ 120 seconds | = 2 880 seconds |
| 24 floor transfers @ 30 seconds | = 720 seconds |
| Total | = 3600 seconds or 1 hour |

Since there are 14 columns of windows per side, it will take 14 hours to wash the windows on one side.

Add in the time required to transfer from one column to the next.

13 transfers between columns @ 60 seconds = 780 seconds (0.217 hours)

Add in the time required to go back to the top.

14 climbs @ 120 seconds = 1 680 seconds (0.467 hours)

| | | |
|---|---|---|
| Total time: | 14 | hours |
| | 0.217 | hours |
| | 0.467 | hours |
| Time for one side: | 14.684 | hours |

Determine the time required to complete all four sides.

$14.684 \text{ hours} \times 4 \text{ sides} = 58.736 \text{ hours}$

$120 \text{ seconds to transfer between sides} \times 3 = 360 \text{ seconds (0.1 hours)}$

Therefore, it will take 58.836 hours to wash all the windows in the building.

**b.** $\frac{58.836}{3} = 19.612$ breaks @ 0.5 hours each is 9.806 hours extra

The washer will need 58.836 + 9.806 = 68.642 hours to complete the project.

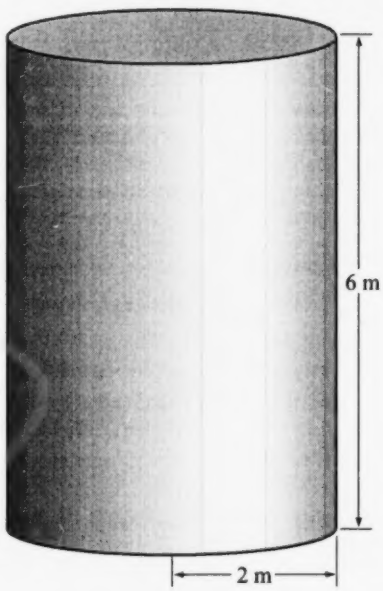
**c.** $120 + 25 \times 68.642 = 1\,836.05$

$1\,836.05 \times 1.10 = 2\,019.66$

The washer should quote the building manager $2 019.66.

*Use the following information to answer the next question.*

A cylindrical storage tank that measures 6 m tall and has a radius of 2 m is going to be covered with sheet metal.




**3.** If sheet metal costs $\$2.08/m^2$, then the cost to cover the entire cylinder, including the top and bottom, will be

**A.** \$75.40
**B.** \$156.83
**C.** \$182.97
**D.** \$209.10

**Solution:**

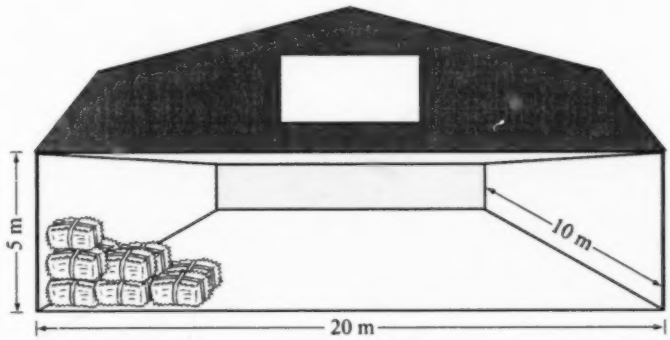
Surface Area of a Cylinder = $2\pi r^2 + 2\pi rh$

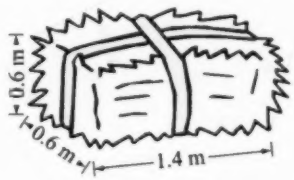
$$SA = 2\pi (2)^2 + 2\pi(2)(6)$$
$$SA = 32\pi$$

Cost of sheet metal = $32\pi$ (\$2.08)
= \$209.10

*Use the following information to answer the next question.*

The usable space in a farm storage shed has the dimensions 20 m × 10 m × 5 m. The straw bales to be stored in the shed each measure 1.4 m × 0.6 m × 0.6 m. The bales are to be stacked in such a way that the long side of the bale runs parallel to the long side of the shed, as shown in the diagram. Bales must not be broken.




**4.** The maximum number of bales that can be stored in this shed is

**A.** 1 120
**B.** 1 792
**C.** 1 982
**D.** 1 984

**Solution:**

$$\frac{20}{1.4} = 14.285...$$

$$\frac{10}{0.6} = 16.66...$$

$$\frac{5}{0.6} = 8.33...$$

One can stack 14 bales across, 16 bales back, and 8 bales up. The maximum number of bales that can be stored in this shed is 1 792.

*Use the following information to answer the next question.*

The surface area to be repainted on an older model car is 17 $m^2$. The paint costs \$42.50 per can, including tax, and each can will cover 2.75 $m^2$. Paint must be purchased in whole cans.

**5.** The total cost to purchase the number of cans of paint required is

**A.** \$297.50

**B.** \$263.50

**C.** \$262.73

**D.** \$240.83

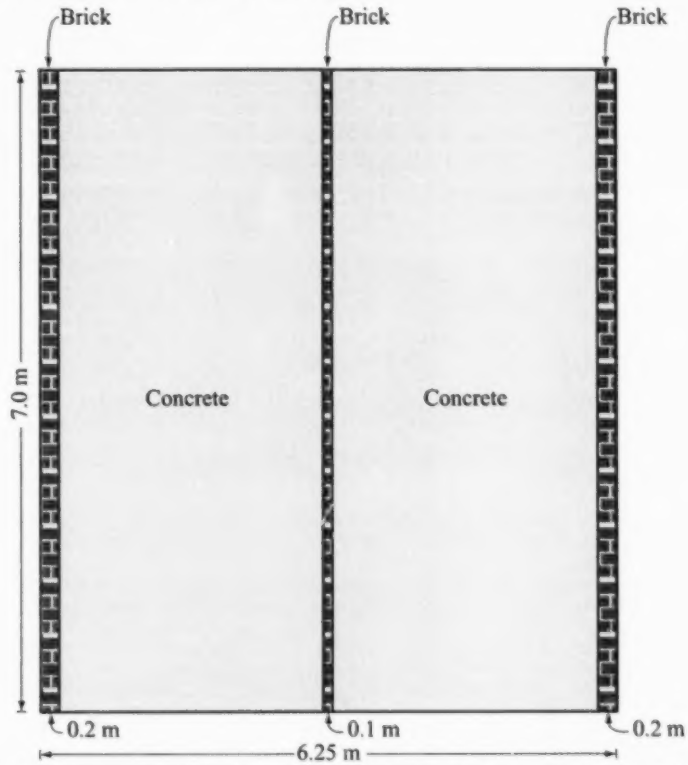
**Solution:**

$\frac{17 m^2}{2.75 m^2 / can} = 6.\overline{18}$ cans of paint are required, so purchase 7 cans of paint.

Each can costs \$42.50, so the total cost to purchase enough paint is \$297.50.

*Use the following information to answer the next question.*

The Hacketts are planning to pour a concrete driveway. The driveway will be 6.25 m by 7.0 m by 0.10 m deep, and the concrete pads will be bordered with brick, as shown in the diagram below.




To determine the cost of the concrete, the Hackett's must first calculate its **volume**.

**6.** If concrete costs \$142.25/m$^3$, the total cost of the concrete portion of the driveway will be

**A.** \$572.56

**B.** \$531.66

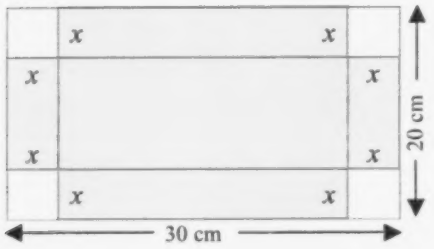
**C.** \$286.28

**D.** \$124.77

**Solution:**

Volume = (6.25 – 0.2 – 0.2 – 0.1)(7.0)(0.10)
Volume = 4.025m$^3$
Total cost = 4.025m$^3$ × \$142.25/m$^3$
= \$572.56

7. A rectangular container is constructed out of a sheet of tin that measures 30 cm × 20 cm. Squares of equivalent sizes are removed from each corner, and then the ends are folded up.




**a.** Determine an equation that could be used to calculate the volume of the container.

**b.** Determine an appropriate window setting to graph this function given the context of this problem. Give reasons for your choice.

**c.** Graph the function in this viewing window and determine the value for $x$ that will give the maximum volume. State the maximum volume.

SE **d.** Determine an expression for the surface area of the box.

SE **e.** The person using the container wants to maximize its volume while keeping the surface area to a relative minimum. Discuss the values for $x$ that you would choose in order to meet these conditions.

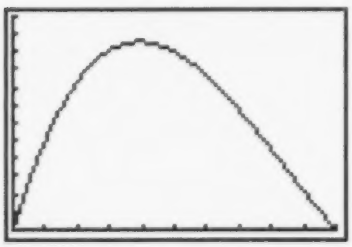
**Solution:**

**a.** $V = x(20 - 2x)(30 - 2x)$

**b.** The values for $x$ relate to the length of the sides of the rectangle. Therefore, 0 cm to 10 cm would be a reasonable choice because the length removed must be smaller than 20 cm to result in a container.

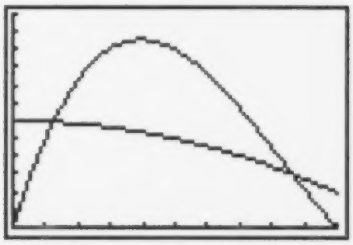
The volume will be greater than 0 $cm^3$ and must be greater than the surface area. A maximum value of 1 200 seems reasonable.
The viewing window should be $x$: [0, 10, 1] and $y$: [0, 1 200, 100].

**c.**



SE **d.** $A = 2x(20 - 2x) + 2x(30 - 2x) + (20 - 2x)(30 - 2x)$

SE **e.** If the function for surface area is graphed on the same grid as the volume function, then there is no minimum surface area.



Since the two functions intersect at $x = 1.23$ and $x = 8.62$, and since the maximum volume is between these two values, any value for $x$ that is between 1.23 cm and 8.62 cm would work well. Values less than 1.23 cm result in the volume being too small, as do values greater than 8.62 cm.

# Applied Mathematics 30 Formula Sheet

## Cost and Design

### Area

Circle $A = \pi r^2$

Triangle $A = \frac{b \times h}{2}$

Parallelogram $A = b \times h$

Trapezoid $A = h\left(\frac{b_1 + b_2}{2}\right)$

### Surface Area

Sphere $SA = 4\pi r^2$

Cylinder $SA = 2\pi r^2 + 2\pi rh$

Cone $SA = \pi r^2 + \pi rs$

### Volume

Sphere $V = \frac{4}{3}\pi r^3$

Cylinder $V = \pi r^2 h$

Prism $V = B \cdot h$, where $B$ is the area of the base

Cone $V = \frac{1}{3}\pi r^2 h$

Pyramid $V = \frac{B \cdot h}{3}$, where $B$ is the area of the base

## Trigonometry and Vectors

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

## Statistics and Probability

$$\mu = np$$

$$\sigma = \sqrt{np(1-p)}$$

$$z = \frac{x - \mu}{\sigma}$$

95% C.I. $= \mu \pm 1.96\sigma$

$P(A \text{ or } B) = P(A) + P(B)$

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

## Regression Models

$$y = a \cdot \sin(bx + c) + d$$

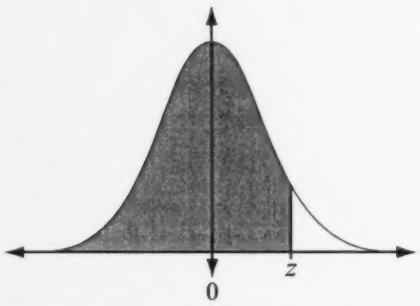
$$\text{period} = \frac{2\pi}{b}$$

$$y = ax^2 + bx + c$$

$$y = ax + b$$

$$y = a \cdot b^x$$

$$z = \frac{x - \mu}{\sigma}$$




***Areas Under the Standard Normal Curve***

| z | 0.09 | 0.08 | 0.07 | 0.06 | 0.05 | 0.04 | 0.03 | 0.02 | 0.01 | 0.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| **–3.4** | 0.0002 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 |
| **–3.3** | 0.0003 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0005 | 0.0005 | 0.0005 |
| **–3.2** | 0.0005 | 0.0005 | 0.0005 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0007 | 0.0007 |
| **–3.1** | 0.0007 | 0.0007 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0009 | 0.0009 | 0.0009 | 0.0010 |
| **–3.0** | 0.0010 | 0.0010 | 0.0011 | 0.0011 | 0.0011 | 0.0012 | 0.0012 | 0.0013 | 0.0013 | 0.0013 |
| **–2.9** | 0.0014 | 0.0014 | 0.0015 | 0.0015 | 0.0016 | 0.0016 | 0.0017 | 0.0018 | 0.0018 | 0.0019 |
| **–2.8** | 0.0019 | 0.0020 | 0.0021 | 0.0021 | 0.0022 | 0.0023 | 0.0023 | 0.0024 | 0.0025 | 0.0026 |
| **–2.7** | 0.0026 | 0.0027 | 0.0028 | 0.0029 | 0.0030 | 0.0031 | 0.0032 | 0.0033 | 0.0034 | 0.0035 |
| **–2.6** | 0.0036 | 0.0037 | 0.0038 | 0.0039 | 0.0040 | 0.0041 | 0.0043 | 0.0044 | 0.0045 | 0.0047 |
| **–2.5** | 0.0048 | 0.0049 | 0.0051 | 0.0052 | 0.0054 | 0.0055 | 0.0057 | 0.0059 | 0.0060 | 0.0062 |
| **–2.4** | 0.0064 | 0.0066 | 0.0068 | 0.0069 | 0.0071 | 0.0073 | 0.0075 | 0.0078 | 0.0080 | 0.0082 |
| **–2.3** | 0.0084 | 0.0087 | 0.0089 | 0.0091 | 0.0094 | 0.0096 | 0.0099 | 0.0102 | 0.0104 | 0.0107 |
| **–2.2** | 0.0110 | 0.0113 | 0.0116 | 0.0119 | 0.0122 | 0.0125 | 0.0129 | 0.0132 | 0.0136 | 0.0139 |
| **–2.1** | 0.0143 | 0.0146 | 0.0150 | 0.0154 | 0.0158 | 0.0162 | 0.0166 | 0.0170 | 0.0174 | 0.0179 |
| **–2.0** | 0.0183 | 0.0188 | 0.0192 | 0.0197 | 0.0202 | 0.0207 | 0.0212 | 0.0217 | 0.0222 | 0.0228 |
| **–1.9** | 0.0233 | 0.0239 | 0.0244 | 0.0250 | 0.0256 | 0.0262 | 0.0268 | 0.0274 | 0.0281 | 0.0287 |
| **–1.8** | 0.0294 | 0.0301 | 0.0307 | 0.0314 | 0.0322 | 0.0329 | 0.0336 | 0.0344 | 0.0351 | 0.0359 |
| **–1.7** | 0.0367 | 0.0375 | 0.0384 | 0.0392 | 0.0401 | 0.0409 | 0.0418 | 0.0427 | 0.0436 | 0.0446 |
| **–1.6** | 0.0455 | 0.0465 | 0.0475 | 0.0485 | 0.0495 | 0.0505 | 0.0516 | 0.0526 | 0.0537 | 0.0548 |
| **–1.5** | 0.0559 | 0.0571 | 0.0582 | 0.0594 | 0.0606 | 0.0618 | 0.0630 | 0.0643 | 0.0655 | 0.0668 |
| **–1.4** | 0.0681 | 0.0694 | 0.0708 | 0.0721 | 0.0735 | 0.0749 | 0.0764 | 0.0778 | 0.0793 | 0.0808 |
| **–1.3** | 0.0823 | 0.0838 | 0.0853 | 0.0869 | 0.0885 | 0.0901 | 0.0918 | 0.0934 | 0.0951 | 0.0968 |
| **–1.2** | 0.0985 | 0.1003 | 0.1020 | 0.1038 | 0.1056 | 0.1075 | 0.1093 | 0.1112 | 0.1131 | 0.1151 |
| **–1.1** | 0.1170 | 0.1190 | 0.1210 | 0.1230 | 0.1251 | 0.1271 | 0.1292 | 0.1314 | 0.1335 | 0.1357 |
| **–1.0** | 0.1379 | 0.1401 | 0.1423 | 0.1446 | 0.1469 | 0.1492 | 0.1515 | 0.1539 | 0.1562 | 0.1587 |
| **–0.9** | 0.1611 | 0.1635 | 0.1660 | 0.1685 | 0.1711 | 0.1736 | 0.1762 | 0.1788 | 0.1814 | 0.1841 |
| **–0.8** | 0.1867 | 0.1894 | 0.1922 | 0.1949 | 0.1977 | 0.2005 | 0.2033 | 0.2061 | 0.2090 | 0.2119 |
| **–0.7** | 0.2148 | 0.2177 | 0.2206 | 0.2236 | 0.2266 | 0.2296 | 0.2327 | 0.2358 | 0.2389 | 0.2420 |
| **–0.6** | 0.2451 | 0.2483 | 0.2514 | 0.2546 | 0.2578 | 0.2611 | 0.2643 | 0.2676 | 0.2709 | 0.2743 |
| **–0.5** | 0.2776 | 0.2810 | 0.2843 | 0.2877 | 0.2912 | 0.2946 | 0.2981 | 0.3015 | 0.3050 | 0.3085 |
| **–0.4** | 0.3121 | 0.3156 | 0.3192 | 0.3228 | 0.3264 | 0.3300 | 0.3336 | 0.3372 | 0.3409 | 0.3446 |
| **–0.3** | 0.3483 | 0.3520 | 0.3557 | 0.3594 | 0.3632 | 0.3669 | 0.3707 | 0.3745 | 0.3783 | 0.3821 |
| **–0.2** | 0.3859 | 0.3897 | 0.3936 | 0.3974 | 0.4013 | 0.4052 | 0.4090 | 0.4129 | 0.4168 | 0.4207 |
| **–0.1** | 0.4247 | 0.4286 | 0.4325 | 0.4364 | 0.4404 | 0.4443 | 0.4483 | 0.4522 | 0.4562 | 0.4602 |
| **–0.0** | 0.4641 | 0.4681 | 0.4721 | 0.4761 | 0.4801 | 0.4840 | 0.4880 | 0.4920 | 0.4960 | 0.5000 |

*Areas Under the Standard Normal Curve*

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| **3.1** | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| **3.2** | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| **3.3** | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| **3.4** | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

## *Explanation of Cognitive Levels*

**Procedures**

The assessment of students' knowledge of mathematical procedures should involve recognition, defence, execution, and verification of appropriate procedures and the steps contained within them. Students must appreciate that procedures are created or generated to meet specific needs in an efficient manner and thus can be modified or extended to fit new situations. The use of technology can allow for conceptual understanding prior to specific skill development. Assessment of students' procedural knowledge will not be limited to an evaluation of their proficiency in performing procedures, but will be extended to reflect the skills presented above.

Certain types of procedural execution cannot be tested on diploma examinations because of restrictions in technology. This procedural execution is, however, an integral part of the *Program of Studies* and should be tested in the classroom.

**Concepts**

An understanding of mathematical concepts goes beyond a mere recall of definitions and recognition of common examples. Assessment of students' knowledge and understanding of mathematical concepts should provide evidence that they can compare, contrast, label, verbalize and define concepts, identify and generate examples and non-examples as well as properties of a given concept, and recognize the various meanings and interpretations of concepts. Students who have developed a conceptual understanding of mathematics can also use models, symbols, and diagrams to represent concepts. Appropriate assessment will also provide evidence of the extent to which students have integrated their knowledge of various concepts.

**Problem Solving**

Appropriate assessment of problem-solving skills is achieved by allowing students to adapt and extend the mathematics they know and by encouraging the use of strategies to solve unique and unfamiliar problems. Assessment of problem solving involves measuring the extent to which students use these strategies and knowledge, and their ability to verify and interpret results. Students' ability to solve problems develops over time as a result of their experience with relevant situations that present opportunities to solve various types of problems.

"Problem solving can be used effectively as a context in which students will learn new concepts and skills." (Hiebert, 1998).

Evidence of problem-solving skills is often linked to clarity of communication. Students demonstrating strong problem-solving skills should be able to clearly explain the process they have chosen using clear language and appropriate mathematical notation and conventions.

# *Calculator Policy and Calculator Clearing Instructions*

"Improvements in technology and its increased availability in schools have changed the focus of mathematics education. The time saved by using calculators and computers to perform complex calculations can be used to help students better understand mathematical concepts. Students can then understand the relationships among concepts and use these relationships to solve problems." (*Applied and Pure Mathematics Program of Studies*, page 9).

It is expected that students writing an Applied Mathematics 30 Diploma Examination will have an approved graphing calculator and be proficient in its use (see *Calculator Policy* for a list of approved calculators). If students are asked to provide a graph in their solution, they will be required to provide the dimensions of the graphing window used. The dimensions are to be communicated in the following manner.

$x$: $[x_{min}, x_{max}, x_{scl}]$ $y$: $[y_{min}, y_{max}, y_{scl}]$

For example, the window format: $x$: [–3, 5, 1] $y$: [–50, 50, 10] would mean the $x$-axis is scaled from –3 to 5 in increments of 1 and the $y$-axis is scaled from –50 to 50 in increments of 10.

Some questions will require the use of a graphing calculator.

E.g., the following table shows the average daily temperatures in Winnipeg, Manitoba, by month over a period of one year.

| **Average Daily Temperature in Winnipeg, Manitoba** | |
|---|---|
| **Month** | **Average Temperature** |
| January | –18 |
| February | –18 |
| March | –11 |
| April | 0 |
| May | 11 |
| June | 18 |
| July | 20 |
| August | 19 |
| September | 14 |
| October | 7 |
| November | –8 |
| December | –17 |

**a.** Use a graphing calculator to plot the data.

**b.** Determine an approximate sinusoidal function for this data. Express the values to the nearest hundredth. Graph this function in the viewing window provided and state the window settings.

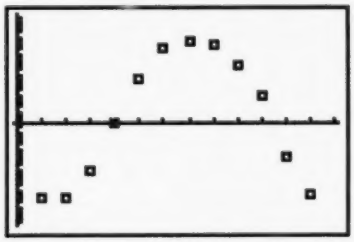
$x:[\quad,\quad,\quad]$

$y:[\quad,\quad,\quad]$

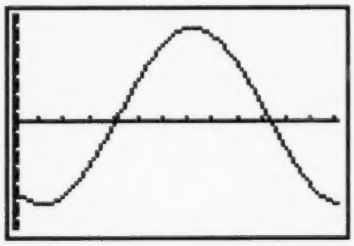
function: _____

**Solution:**

**a.** A possible solution:



**b.**



$x:[\ 0,\ 13,\ 1\ ]$

$y:[\ -25,\ 25,\ 1\ ]$

function: $y = 20.61 \sin(0.52x - 2.16) + 1.21$

# Alberta Learning Calculator Policy for Diploma Examinations

Updated September 2002

## Policy

Pure Mathematics 30 and Applied Mathematics 30 require the use of an **approved graphing calculator** for diploma examinations. All science diploma examinations and the Mathematics 33 Diploma Examination require the use of a scientific calculator or an approved graphing calculator. School authorities are required to ensure that students taking Pure Mathematics 30 or Applied Mathematics 30 have access to an approved graphing calculator for diploma examinations

## Definition

The calculator must be a hand-held device designed primarily for mathematical computations, including logarithmic and trigonometric functions, as well as for graphing functions. Included in this definition are those scientific calculators having graphing and programmable features.

## Rationale

Any scientific calculator will be adequate for any science diploma examination. Many students will use an approved graphing calculator, which is also sufficient for science diploma examinations.

The student learning outcomes of the *Information and Communication Technology (K-12) Program of Studies (2000–2003) are being infused into all subject areas.* The *Mathematics Applied and Pure Programs of Study* state:

> *Improvements in technology and its increased availability in schools have changed the focus of mathematics education. The time saved by using calculators or computers to perform complex calculations can be used to help students better understand mathematical concepts. Students can then understand the relationships among concepts and use these relationships to solve problems.*
>
> *Calculators and computers can be used as tools to*
> - *explore and demonstrate mathematical relationships and patterns*
> - *organize and display data*
> - *assist with solving problems*
> - *decrease the time spent on tedious computations*
> - *simulate situations*

To ensure compatibility with the provincial *Programs of Study* and equity and fairness to all students, Alberta Learning expects students to use calculators, as defined, when writing diploma examinations in mathematics and the sciences. Students must not use unapproved graphing calculators for diploma examinations because to do so may confer an undue advantage or undue disadvantage on the student. For example, the use of an older, previously but not currently approved graphing calculator for a mathematics diploma examination will disadvantage the student through the calculator's inability to perform functions required by the *Mathematics Applied and Pure Programs of Study.*

Professional mathematics organizations such as the National Council of Teachers of Mathematics (NCTM) have strongly endorsed the use of graphing calculators in mathematical instruction. They also emphasize that if calculator and computer technologies are now accepted as part of the environment in which students learn and do mathematics, such tools should also be available to students in most assessment situations.

**The *Programs of Study* for the Pure Mathematics 30 and Applied Mathematics 30 mandate the use of graphing calculators. Although many questions on the mathematics diploma examinations will not require the use of a graphing calculator, there will be some questions for which the use of an approved graphing calculator will be required.**

**Calculators that do not meet the minimum properties, such as the TI-81, the TI-82, the TI-85, and scientific calculators, will disadvantage students who may wish to use them on a Pure Mathematics 30 or Applied Mathematics 30 diploma examination and are therefore not approved for diploma examination use.**

**Note:** This policy applies to Alberta Learning diploma examinations and field tests only. School calculator policies may differ depending on grade level or topic studied.

## Expectations

1. At the beginning of **any** mathematics or science diploma examination course, teachers **must** advise students in writing of the types of calculators approved by Alberta Learning for use when writing diploma examinations in these courses.

2. Students must clear **all** programmable calculators, both graphing and scientific, that are brought into diploma examinations of all information that is stored in the programmable or parametric memory.

3. Students must not bring external devices (peripherals) to support calculators into any examination. Such devices include manuals, printed or electronic cards, printers, memory expansion chips or cards, external keyboards, CD-ROMs, libraries, or any annotations that outline operational procedures.

4. In preparation for calculator failure, students may bring extra batteries and/or approved calculators into the examination room.

5. During examinations, supervising teachers must ensure that
   - calculators operate in silent mode
   - students do not share calculators or information contained within them
   - calculator cases are not available to students
   - programmable calculator memories, including parametric memories, have been cleared
   - only graphing calculators on the current list approved by Alberta Learning are used

## Calculator Criteria

The following criteria will be used to select acceptable calculators.

**Minimum calculator properties required**

1. Function graphing capabilities with display
   —includes displaying more than one function on the screen at a time, tracing a function
2. Standard scientific calculator operations
   —e.g., sine, cosine, tangent, inverse functions, logarithms, power ($x^n$)
3. Statistical functions in 1 and 2 variables
   —mean, median, mode, standard deviation, bivariate data
4. Regression models
   —linear, quadratic, exponential, sinusoidal
5. List capabilities
6. Matrix capabilities
   —scalar multiplication, addition, and subtraction

**Unacceptable calculator properties during examinations**

1. Built-in notes (definitions or explanations in alpha notation), e.g., libraries
2. Upgrades/downloads that include built-in notes or formulas

The following list of approved calculators is provided to assist students and teachers in the selection of graphing calculators that conform to the requirements stated in the definition and to the stated criteria. The list will be updated annually.

*Note: All the calculators listed below meet the "required properties." They do not have any "unacceptable properties" and so can be used on the mathematics and science diploma examinations. However, students and teachers should recognize that the different models of calculators listed have a range of capabilities, and the choice of which model to use or purchase will require personal or teacher analysis of the machines' capabilities and one's individual or school circumstances.*

## The List of Approved Graphing Calculators for 2002–2003

| *Brands* | *Casio* | *Sharp* | *Hewlett-Packard* | *Texas Instruments* |
|---|---|---|---|---|
| *Models* | Algebra FX 2.0<br>Algebra FX 2.0 Plus | EL-9600C | HP39G* | TI-83 Plus<br>TI-83 Plus Silver<br>TI-86<br>TI-89<br>TI-92 Plus |

*** The HP39G calculator will remain on the approved list for the 2002–2003 school year and then will be deleted from the approved list. This calculator is not recommended for students entering high school mathematics who wish to use a calculator throughout their high school program.**

The following calculators meet the graphing calculator criteria and are approved, but are no longer commercially manufactured.

| *Brands* | *Casio* | *Sharp* | *Texas Instruments* |
|---|---|---|---|
| *Models* | FX-9700 series<br>CFX-9800G<br>CFX-9850G<br>CFX-9850GA Plus | EL-9600<br>EL-9200<br>ElL9300C | TI-83<br>TI-92 |

*Note: Instructions for clearing calculator memories are outlined on pages 133 to 136 of this document and are also posted on the Alberta Learning web site:*
*http://www.learning.gov.ab.ca/K_12/testing/diploma/bulletins/essential/clearing_calc.asp*

### For Further Information

If you have any questions or comments about this policy, please contact Dr. John Rymer, Mathematics/Science Diploma Examination Unit, Learner Assessment Branch, at (780) 427-0010, fax (780) 422-4454. To call toll-free from outside of Edmonton, dial 310-0000.

## Procedures to be followed prior to the writing of a diploma examination.

1. Students may use any type of calculator approved by Alberta Learning when writing diploma examinations.

2. Students must clear **all** programmable calculators, both graphing and scientific, that are brought into diploma examinations. All information that is stored in the programmable or parametric memory must be cleared.

3. Presiding examiners are responsible for ensuring that
   a. all calculators operate in silent mode
   b. students do not share calculators or information contained within them
   c. calculator cases are not available to students
   d. all examination rules are followed

**Notes:**

- Resetting calculators may result in altering the calculator mode settings. Please remind students to check the mode settings before proceeding with the diploma examination.
- Programs downloaded from the web are not allowed on the calculators used during diploma examinations. These programs must be erased before the student is allowed to write the examination.
- The memory values given on the next pages refer to memory expected to be available as a factory setting. The values available in student calculators should match these values when the calculator has been reset. If the memory values in the student calculators do not match these values, or are not within 1000 of the suggested factory settings, then the calculators should be reset a second time. If this fails to change the values, then the calculator should not be used on the examination.

## Keystrokes Required For Clearing **Approved** Calculators

| **Texas Instruments** | **Memory remaining** |
|---|---|
| TI 83<br>$2^{nd}$ + (MEM)<br>**5** (Reset)<br>**1** (All memory)<br>**2** (Reset)<br>**Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use both $2^{nd}$ and ⇑ repeatedly.** | $2^{nd}$ + **RAM 27118**<br>**1** |
| TI 83 Plus **or** TI 83 Plus Silver Edition<br>$2^{nd}$ + (MEM)<br>**7** (Reset)<br>**➤➤** (All)<br>**Enter**<br>**2** (Reset)<br>**The application "Finance" is the only one that remains after following the clearing instructions.**<br>**Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use both $2^{nd}$ and ⇑ repeatedly.** | TI 83 Plus<br>$2^{nd}$ + **RAM 24317**<br>**2** **ARC 163840**<br>TI 83 Plus Silver Edition<br>$2^{nd}$ + **RAM 24317**<br>**2** **ARC 1540K** |
| TI 86<br>$2^{nd}$ **3** (MEM menu)<br>**F3** (Reset)<br>**F1** (All)<br>**F4** (Yes)<br>**Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use both $2^{nd}$ and ⇑ repeatedly.** | $2^{nd}$ **3** **MEM FREE 98226**<br>**F1** |
| TI 89<br>$2^{nd}$ **6** (MEM)<br>**F1** (All)<br>**1** (Reset)<br>Enter | $2^{nd}$ **6** **RAM 199154**<br>**ARC 393204** |
| TI 92 **or** TI 92 Plus<br>$2^{nd}$ **6** (MEM)<br>**F1** (Reset)<br>**1** (All)<br>**Enter**<br>**Note: If, on clearing, the screen is blank, the contrast needs to be reset. To do this, use ◊ (green) and + or – repeatedly.** | $2^{nd}$ **6** **System 61064**<br>**Memory Free 70008** |

## Casio

| | Memory remaining |
|---|---|
| Casio FX9700 series<br>Go to **Menu**<br>Cursor to **Reset**<br>**EXE** (All Memory)<br>**F1** (Yes – Reset All) | **Menu** **24 000 Bytes available**<br>**EXE**<br>**Clear**<br>**Shift**<br>**MDISP** (CAPA) |
| Casio CFX9800G<br>Go to **Menu**<br>**Options** **EXE** (Memory)<br>**Reset** **EXE** (Reset)<br>**F1** (Yes – Reset All) | **Menu** **24 000 Bytes available**<br>**EXE**<br>**Clear**<br>**Shift**<br>**MDISP** (CAPA) |
| Casio CFX-9850G, CFX9850GA Plus<br>Go to **Menu**<br>Cursor to **Mem**<br>**EXE** (Memory)<br>⇓ (Reset)<br>**EXE**<br>**F1** (Yes – Reset All)<br>**Note: For the GA plus, use Alpha F.** | **Menu** **30677**<br>**Alpha E**<br>**EXE**<br>**Check usage** |
| Casio Algebra FX 2.0 **or** Algebra FX 2.0 Plus<br>Go to **Menu**<br>Cursor to **SYSTEM**<br>**EXE** System manager<br>**F5** Reset<br>**F2** Reset (Clear) Main Memories?<br>**EXE** Yes – Main Memories Cleared<br>**ESC** Reset<br>**F3** Reset (Clear) Storage Memories?<br>**EXE** Test – Storage Memories Cleared<br>**Menu**<br>**Note: A tutorial system exists on this calculator and may be locked out for 180 minutes by following steps illustrated in the manual. Since the information in the tutorial has no effect on the Pure 30/Applied 30 Mathematics topics, application of this process is not necessary. This may have implications for Pure 10-20/Applied 10-20 topics.** | Go to **Menu**<br>Cursor to **Memory**<br>**EXE** **Memory Manager**<br>**F1** **Current Area No Program (146912 Bytes Free)**<br>**F6** **Storage Area No Program (786240 Bytes Free)**<br>**Menu** |

## Sharp

| | Memory remaining |
|---|---|
| Sharp EL 9200, 9300 (or 9300C), 9600, and 9600 C<br>**2nd** **XθTN** (Option)<br>**Log** (Reset)<br>**2** (All Memory)<br>**CL** (Clear all data)<br>**Note: There is also a reset switch on the back. (Use round tip of pen, press, then CL)** | **2nd** ⇓ **XθTN** **18562** |

## Hewlett-Packard

| | Memory remaining |
|---|---|
| Hewlett-Packard HP39G<br>Hold down simultaneously:<br>**ON** Menu Key 1 Menu Key 6<br>Release<br>**OK** (Menu Key 6) | **SHIFT** › **Memory Manager 234 K**<br>**Note: All items in this window should read 0 KB.** |

If you find that there are problems with any of the clearing techniques, please contact the Director of the Mathematics/Science Diploma Examination Unit, Learner Assessment Branch, at (780) 427-0010, fax (780) 422-4454. To call toll-free from outside of Edmonton, dial 310-0000.

## *Mathematics and Science Directing Words*

| | |
|---|---|
| **Discuss** | The word "discuss" **will not** be used as a directing word on math and science diploma examinations because it is not used consistently to mean a single activity. |
| | ***The following words are specific in meaning.*** |
| **Algebraically** | Using mathematical procedures that involve letters or symbols to represent numbers |
| **Analyze** | To make a mathematical, chemical, or methodical examination of parts to determine the nature, proportion, function, interrelationship, etc. of the whole |
| **Compare** | Examine the character or qualities of two things by providing characteristics of both that point out their mutual *similarities* and *differences* |
| **Conclude** | State a logical end based on reasoning and/or evidence |
| **Contrast/Distinguish** | Point out the *differences* between two things that have similar or comparable natures |
| **Criticize** | Point out the *merits* and *demerits* of an item or issue |
| **Define** | Provide the essential qualities or meaning of a word or concept; make distinct and clear by marking out the limits |
| **Describe** | Give a written account or represent the characteristics of something by a figure, model, or picture |
| **Design/Plan** | Construct a plan, i.e, a detailed sequence of actions, for a specific purpose |
| **Determine** | Find a solution, to a specified degree of accuracy, to a problem by showing appropriate formulas, procedures, and calculations |
| **Enumerate** | Specify one by one or list in concise form and according to some order |
| **Evaluate** | Give the significance or worth of something by identifying the good and bad points or the advantages and disadvantages |
| **Explain** | Make clear what is not immediately obvious or entirely known; give the cause of or reason for; make known in detail |
| **Graphically** | Using a drawing that is produced electronically or by hand, and that shows a relation between certain sets of numbers. |
| **How** | Show in what manner or way, with what meaning |

| | |
|---|---|
| **Hypothesize** | Form a tentative proposition intended as a possible explanation for an observed phenomenon; i.e., a possible cause for a specific effect. The proposition should be testable logically and/or empirically |
| **Identify** | Recognize and select as having the characteristics of something |
| **Illustrate** | Make clear by giving an example. The form of the example must be specified in the question; i.e., word description, sketch, or diagram |
| **Infer** | Form a generalization from sample data; arrive at a conclusion by reasoning from evidence |
| **Interpret** | Tell the meaning of something; present information in a new form that adds meaning to the original data |
| **Justify/Show How** | Show reasons for or give facts that support a position |
| **Model** | Find a model (in mathematics, a model of a situation is a pattern that is supposed to represent or set a standard for a real situation) that does a good job of representing a situation |
| **Outline** | Give, in an organized fashion, the essential parts of something. The form of the outline must be specified in the question; i.e., lists, flow charts, concept maps |
| **Predict** | Tell in advance on the basis of empirical evidence and/or logic |
| **Prove** | Establish the truth of validity of a statement for the general case by giving factual evidence or logical argument |
| **Relate** | Show logical or causal connection between things |
| **Sketch** | Provide a drawing that represents the key features of an object or graph |
| **Solve** | Give a solution for a problem; i.e., explanation in words and/or numbers |
| **Summarize** | Give a brief account of the main points |
| **Trace** | Give a step-by-step description of the development |
| **Verify** | Establish, by substitution for a particular case or by geometric comparison, the truth of a statement |
| **Why** | Show the cause, reason, or purpose |